THEORETICAL ANALYSIS OF TRANSONIC FLOW PAST UNSTAGGERED OSCILLATING CASCADES

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THESIS

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TRANSONIC FLOW PAST
UNSTAGGERED OSCILLATING CASCADES

by

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Theoretical Analysis of Transonic Flow Past Unstaggered Oscillating Cascades

by

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ABSTRACT

This paper presents an independent verification of the collocation method as a technique for calculating the lift on an oscillating airfoil in an unstaggered cascade immersed in transonic flow. This method was originally proposed by Gorelov. Results presented here differ somewhat from those presented by him. Two formulations are shown; one is purely numerical, the second employs an analytic expansion for small frequency.

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LIST OF SYMBOLS

a	=	local speed of sound	II
a_o	=	speed of sound in the uniform flow	III
c	=	blade semichord	III
f_j	=	elementary function used in collocation solution	V
F	=	specific energy ("head")	II
G	=	function describing the surface of the airfoil as a function of time	III
H	=	function specifying location of blade surface in the vertical axis	III
i	=	$\sqrt{-1}$	II,III,IV,V
k	=	Strouhal number, nondimensional frequency	III,IV,V
m	=	$\sqrt{(\gamma+1)w}$	IV,V
M	=	Mach number = $\frac{\lvert\vec{V}\rvert}{a}$	III
n	=	number of collocation points - 1, order of highest spanning function	V
p	=	pressure	II
	=	nondimensional interblade distance	V
R	=	universal gas constant	II
R.P.	=	"real part of"	III,IV,V
T	=	temperature	II
t	=	time, nondimensional time	II,III,IV,V
U_o	=	uniform velocity from infinity	II,III,VI
u	=	x-component of velocity	II,III
u^o, u^1	=	interference vertical velocities due to reference and adjacent blades respectively, solved so as to satisfy the tangential flow conditions	V,VI

u'	=	small disturbance velocity	III
v	=	y-component of velocity	II,III,IV,V
$\vec{V}$	=	general velocity vector	II
v'	=	small disturbance velocity	III
v^o, v^1	=	vertical velocities due to the reference and adjacent blades respectively, determined from the tangential flow condition	V
w	=	$\tilde{\phi}_x$, a constant used in Gorelov's approximation of the transonic flow potential	IV,V
x	=	horizontal coordinate, may be non-dimensional	
x_*	=	mp (transformed interblade distance in Gorelov's approximation)	V
x_ℓ	=	blade leading edge	IV
x_o	=	center of pitch of the unstaggered cascade	IV,V
y	=	vertical coordinate, may be non-dimensional	
y, y_1	=	vertical coordinates attached to the reference and adjacent blades respectively, may be non-dimensional	IV,V
z, z_1	=	transformed vertical coordinates used in Gorelov's approximation, attached to the reference and adjacent blades respectively. $z = my$, $z_1 = my_1$	IV,V
$\frac{D}{Dt}$	=	substantial derivative w.r.t. time $= \frac{\partial}{\partial t} + \frac{\partial x}{\partial t}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial}{\partial y}$ $= \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$	II,III
$O(\omega^2)$	=	"of the order of magnitude of ω^2"	V
α	=	angle of attack	II,III,IV,V
α_o	=	maximum amplitude of pitch oscillations	IV

γ	=	ratio of specific heats, c_p/c_v	
δ_{io}	=	Dirac Delta function $= \begin{matrix} 1 & \text{when} & i = 0 \\ 0 & \text{when} & i \neq 0 \end{matrix}$	V
η	=	$\cos^{-1}(-x)$	V
η_*	=	$\cos^{-1}(1-x_*)$	V
$\bar{\eta}$	=	$\cos^{-1}(-s)$	V
$\hat{\eta}$	=	$\cos^{-1}(x_*-x)$	V
θ_j^o,θ_j^1	=	interference potential coefficients for reference and adjacent blades respectively	V
λ	=	k/m^2	IV,V
μ_j^o,μ_j^1	=	Fourier coefficients describing the motion of the reference and adjacent blades respectively	V
ν	=	angular frequency of oscillation	IV,V
ρ	=	density	II
σ	=	phase angle	V
τ	=	cascade solidity, $\frac{2}{p}$	V
Φ	=	general velocity potential	II,III,VI
Φ_o	=	uniform flow velocity potential	
$\tilde{\phi}$	=	steady flow perturbation potential	III,IV
ϕ^o,ϕ^1	=	perturbation potential in collocation solution due to reference and adjacent blades respectively	V,VI
Φ^o,Φ^1	=	tranformed potentials	V
ψ	=	oscillatory flow potential	III,IV,V
Φ	=	transformed oscillatory potential in Gorelov's coordinates corresponding to ψ	V
ψ^o,ψ^1	=	interference potentials due to reference and adjacent blades respectively	V,VI

Ψ^o, Ψ^1 = transformed potentials in Gorelov's coordinates, coresponding to ψ^o and ψ^1 V

ω = $\dfrac{k(1-m)^2}{m^4}$

∇ = $\vec{i}\,\dfrac{\partial}{\partial x} + \vec{j}\,\dfrac{\partial}{\partial y}$, Gradient operator, $\vec{i}$ and $\vec{j}$ are unit vectors in the x and y direction respectively

Computer Variables

DK	=	Reduced Frequency, k
DLAMDA	=	λ
DM2	=	m^2
DR	=	mp = x_*
ETA	=	η
ETASTR	=	η_*
IPT	=	Print Parameter
N	=	n
NF	=	not used
OFFSET	=	r
OMEGA	=	ω
QALPHA	=	$0 + i(\lambda - k)$
QCONST	=	$e^{i\sigma}$
QDCL	=	$C\ell_{\alpha}$
QDCM	=	Cm_{α}
QDK	=	$0 + ik$
QEXP	=	$0 + i\lambda$
QINTAP,QINTRP	=	Variables used to transmit boundary condition integrals
Q1ABCF,Q1RBCF	=	Interference coefficients for adjacent and reference blades
Q1COF	=	Right hand side vector in collocation solution
Q1INT	=	Known matrix of integrals in collocation solution
Q1RBP,Q1ABP	=	Not used
Q2CP	=	Not used
Q2EXP	=	$e^{-i\lambda x}$

Q2PT	=	Not used
RHO	=	vertical displacement
RHP	=	local input variable for rho
SIGMA	=	σ
TAU	=	τ
XASTN	=	Current x station in adjacent blade coordinates
XSTN	=	Current x station in reference blade coordinates

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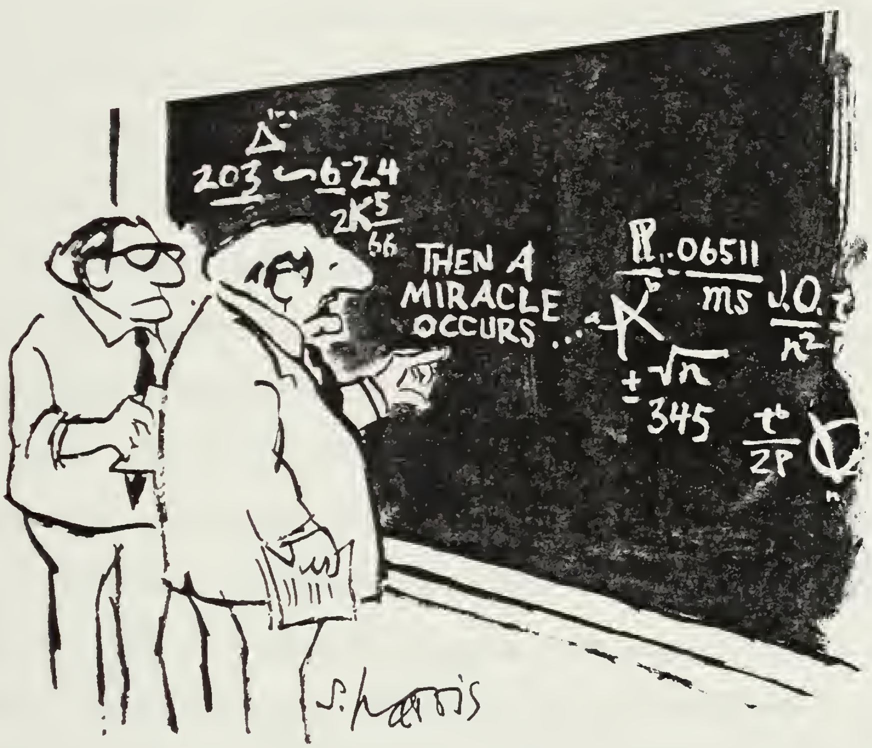

"I think you should be more explicit here in step two."

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I. INTRODUCTION

The analysis of unsteady transonic flows in aircraft turbopropulsion is an area of intense current interest. Rising fuel prices and increasing thrust requirements both point toward the need of turbomachinery capable of performing well with transonic or supersonic internal flow. But, increased flow has increased both the costs and uncertainties of engine designs. Flutter problems have already become a major consideration in engine development. Problems unforeseen in earlier days of turbine engine production have caused long development delays, or forced acceptance of engines producing less than their initial design thrust. These uncertainties cannot be avoided when an attempt is made to extend the state of the art, but they can be reduced by extending the range of analytical modeling.

Such extension must now be done piecemeal. The three-dimensional flows in turbomachinery, including the simultaneous effects of boundary layers, rotation, finite blade thickness, spanwise Mach distributions, and shocks, are well beyond present capability. Perhaps one day complete analysis will be practical, but it is not today. The best that can be done now is to approach the problem from one aspect at a time. Flow through a two dimensional cascade has been a useful tool in this partial analysis.

This thesis was originally to have been an extension of the work of Elder [1] and Schlein [2] to the case of a staggered cascade. Their work, based on Teipel's [3] linearization of the unsteady transonic small perturbation equation, analyzed transonic flow through oscillating unstaggered cascades by use of the collocation method. While the problem was easy to state, it was difficult to solve. Both Elder and Schlein had encountered difficulty in employing the collocation method. Therefore, it was decided that verification of the basic collocation solution presented by Gorelov [4] using a different linearization would be a worthwhile goal in itself.

The following investigation presents a verification of the development in [4], along with numerical results and suggestions for further work.

II. UNSTEADY TRANSONIC FLOW THEORY

Considering inviscid flow only, the following four equations govern the aerodynamic flow problem at hand:

The equation of state

$$p = \rho RT \qquad \text{(II-1)}$$

and the equations for the conservation of

1. Mass: $$\operatorname{div}(\rho\vec{v}) + \frac{\partial \rho}{\partial t} = 0 \qquad \text{(II-2)}$$

2. Momentum: $$\frac{D\vec{v}}{Dt} + \frac{1}{\rho}\nabla p = 0 \qquad \text{(II-3)}$$

3. Energy: $$\frac{DS}{Dt} = 0 \qquad \text{(II-4)}$$

where

$\vec{v}$ = velocity

p = pressure

S = entropy

R = universal gas constant

T = temperature

t = time

ρ = density

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial}{\partial y}\frac{\partial y}{\partial t}$$

The analysis starts with a uniform flow from infinity. This flow has velocity U_o parallel to the x-axis. This

formulation can be simplified by working with the total velocity potential, Φ, where

$$u = \frac{\partial \Phi}{\partial x} = \Phi_x = x \text{ component of velocity} = \frac{\partial x}{\partial t} \qquad \text{(II-5)}$$

$$v = \frac{\partial \Phi}{\partial y} = \Phi_y = y \text{ component of velocity} = \frac{\partial y}{\partial t} \qquad \text{(II-6)}$$

Thus, the initial uniform flow is represented by the uniform flow potential

$$\Phi_o = U_o x \qquad \text{(II-7)}$$

This notation may be applied to the conservation equations for mass and momentum. The equation for conservation of mass

$$\operatorname{div}(\rho\vec{v}) + \frac{\partial \rho}{\partial t} = 0 \qquad \text{(II-8)}$$

becomes for two-dimensional unsteady flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial \rho}{\partial t} = 0$$

$$\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right] + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad \text{(II-9)}$$

but

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} = \frac{D\rho}{Dt}$$

and

$$u = \Phi_x \quad \text{and} \quad v = \Phi_y$$

Thus

$$\frac{D\rho}{Dt} + \rho(\Phi_{xx} + \Phi_{yy}) = 0$$

and

$$\Phi_{xx} + \Phi_{yy} = -\frac{1}{\rho}\frac{D\rho}{Dt} \qquad \text{(II-10)}$$

The speed of sound is given by

$$a^2 = \frac{dp}{d\rho}$$

Thus

$$\frac{D\rho}{Dt} = \frac{d\rho}{dp} \cdot \frac{Dp}{Dt} = \frac{1}{a^2}\frac{Dp}{Dt}$$

Applying this to equation (II-10) yields

$$\Phi_{xx} + \Phi_{yy} = -\frac{1}{\rho a^2}\frac{Dp}{Dt} \qquad \text{(II-11a)}$$

$$\Phi_{xx} + \Phi_{yy} = -\frac{1}{\rho a^2}(up_x + vp_y + p_t) \qquad \text{(II-11b)}$$

$$= -\frac{1}{\rho a^2}[(\nabla\Phi)\cdot(\nabla p) + p_t] \qquad \text{(II-11c)}$$

where ∇ is the gradient operator, $P_x = \frac{\partial P}{\partial x}$, $P_y = \frac{\partial P}{\partial y}$, $P_t = \frac{\partial P}{\partial t}$.

Laying this aside for the moment, consider the momentum equation (II-3)

$$\frac{D\vec{v}}{Dt} + \frac{1}{\rho} \nabla p = 0$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v}$$

Thus

$$\rho [\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v}] = \nabla p \tag{II-12}$$

$$\vec{v} = \nabla \Phi$$

so

$$\frac{\partial \vec{v}}{\partial t} = \frac{\partial}{\partial t}(\nabla \Phi) = \nabla \frac{\partial \Phi}{\partial t} \tag{II-13}$$

and

$$(\vec{v} \cdot \nabla) \vec{v} = \nabla \frac{V^2}{2} - \vec{v} \times (\nabla \times \vec{v})$$

where

$$V^2 = u^2 + v^2$$

$$= \vec{v} \cdot \vec{v}$$

For irrotational flow

$$\nabla \times \vec{v} = 0$$

thus

$$(\vec{v} \cdot \nabla)\, \vec{v} = \frac{\nabla V^2}{2} \qquad \text{(II-14)}$$

Thus

$$\frac{\nabla p}{\rho} + \nabla [\Phi_t + \frac{V^2}{2}] = 0 \qquad \text{(II-15)}$$

which after integration along a streamline becomes

$$\int \frac{dp}{\rho} + \Phi_t + \frac{V^2}{2} = F(t) \qquad \text{(II-16)}$$

For uniform flow from infinity $F(t) = \frac{1}{2} U_o{}^2$ and thus the final result is

$$\int \frac{dp}{\rho} + \Phi_t + \frac{V^2}{2} = \frac{1}{2} U_o{}^2 \qquad \text{(II-17)}$$

Differentiation with respect to t gives

$$p_t = -\rho(\Phi_{tt} + \frac{1}{2}\frac{\partial V^2}{\partial t}) \tag{II-18}$$

From (II-3)

$$-\nabla p = \rho \frac{D\vec{v}}{Dt} \tag{II-19}$$

Substitute (II-18) and (II-19) into (II-11c) to obtain

$$\Phi_{xx} + \Phi_{yy} = \frac{1}{a^2}[(\nabla\Phi) \cdot \frac{D\vec{v}}{Dt} + \Phi_{tt} + \frac{1}{2}\frac{\partial V^2}{\partial t}] \tag{II-20}$$

This may be further simplified

$$\frac{D\vec{v}}{Dt} = \frac{\partial\vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\,\vec{v}$$

$$= \frac{\partial\vec{v}}{\partial t} + \frac{1}{2}\nabla V^2$$

Hence

$$\Phi_{xx} + \Phi_{yy} = \frac{1}{a^2}[(\nabla\Phi)\cdot(\frac{\partial\vec{v}}{\partial t} + \frac{1}{2}\nabla V^2) + \Phi_{tt} + \frac{1}{2}\frac{\partial V^2}{\partial t}] \tag{II-21}$$

Expanding terms

$$(\nabla\Phi) \cdot (\frac{\partial\vec{v}}{\partial t}) = \nabla\Phi \cdot (\frac{\partial}{\partial t}\nabla\Phi) = \Phi_x\Phi_{xt} + \Phi_y\Phi_{yt}$$

$$\nabla\Phi \cdot \frac{\nabla V^2}{2} = \frac{\Phi_x{}^2\Phi_{xx}}{2} + \frac{\Phi_y{}^2\Phi_{yy}}{2} + \frac{\Phi_x\Phi_y\Phi_{xy}}{2} + \frac{\Phi_y\Phi_x\Phi_{yx}}{2}$$

$$= \frac{\Phi_x{}^2\Phi_{xx}}{2} + \frac{\Phi_y{}^2\Phi_{yy}}{2} + \frac{2\Phi_x\Phi_y\Phi_{xy}}{2} .$$

$$\frac{1}{2}\frac{\partial V^2}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}[(\nabla\Phi) \cdot (\nabla\Phi)]$$

$$= \frac{1}{2}[\Phi_x\Phi_{xt} + \Phi_{xt}\Phi_x + \Phi_y\Phi_{yt} + \Phi_{yt}\Phi_y]$$

$$= \Phi_x\Phi_{xt} + \Phi_y\Phi_{yt}$$

The final result obtained by combining terms is

$$\left(1 - \frac{\Phi_x^{\,2}}{a^2}\right){}_{xx} + \left(1 - \frac{\Phi_y^{\,2}}{a^2}\right){}_{yy} - \frac{2\Phi_x\Phi_y\Phi_{xy}}{a^2}$$

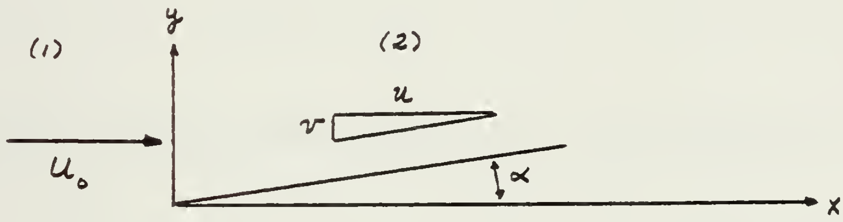
$$- \frac{2\Phi_x}{a^2}\Phi_{xt} - \frac{2\Phi_y}{a^2}\Phi_{yt} - \frac{1}{a^2}\Phi_{tt} = 0 \qquad \text{(II-22)}$$

This result is valid for irrotational, inviscid, two-dimensional, unsteady, compressible flows where gravity has been neglected.

III. SMALL PERTURBATION THEORY OF TRANSONIC FLOW

A. GENERAL CASE

A thin body at a small angle of attack will cause only a slight disturbance in the fluid. A flat plate is an example. Consider flow past a flat plate at angle of attack, α.



The flow at (2) must be parallel to the plate. To achieve this, small disturbance velocities u' and v' must be added to the free stream velocity yielding

$$u = U_o + u'$$

$$v = v'$$

The potential of the disturbed flow may be considered as the sum of the uniform flow potential, $\Phi_o = U_o x$, and the disturbance potential, ϕ

$$\Phi = \Phi_o + \phi \tag{III-1}$$

Thus

$$\Phi_x = U_o + u' \tag{III-2a}$$

$$\Phi_y = v \tag{III-2b}$$

If ϕ is a function of time, then

$$\Phi_t = \phi_t$$

This result may be substituted into (II-22) leading to

$$[1 - \frac{(U_o + u')^2}{a^2}]\phi_{xx} + [1 - \frac{v^2}{a^2}]\phi_{yy} - 2\frac{(U_o + u')v}{a^2}\phi_{xy}$$

$$- 2\frac{(U_o + u')}{a^2}\phi_{xt} - \frac{2v}{a^2}\phi_{yt} - \frac{1}{a^2}\phi_{tt} = 0 \tag{III-3}$$

This expands to yield

$$[1 - \frac{U_o^2 + 2U_o u' + u'^2}{a^2}]\phi_{xx} + [1 - \frac{v^2}{a^2}]\phi_{yy} - 2\frac{(U_o v + u'v)}{a^2}\phi_{xy}$$

$$- 2\frac{(U_o + u')}{a^2}\phi_{xt} - \frac{2v}{a^2}\phi_{yt} - \frac{1}{a^2}\phi_{tt} = 0 \tag{III-4}$$

Equation (III-4) may be further simplified as shown by Landahl [3]. All non-linear terms except the $\phi_x\phi_{xx}$ product

term can be neglected yielding the following transonic small disturbance equation

$$[(M^2-1) + (\gamma+1)M^2 \frac{\phi_x}{U_o}]\phi_{xx}$$

$$- \phi_{yy} + \frac{2M_o}{a_o} \phi_{xt} + \frac{1}{a_o} \phi_{tt} = 0 \qquad \text{(III-5)}$$

where a_o is the velocity of sound in the free-stream, and γ is the ratio of specific heats, and M = Mach number.

B. BOUNDARY CONDITION

The tangential flow condition requires that the flow be tangent to the airfoil surface at each instant of time. This means that no fluid may flow through the surface of the airfoil and is expressed by the condition

$$\frac{DG}{Dt} = 0 \quad \text{on} \quad G(x,y,t) \qquad \text{(III-6)}$$

where

G(x,y,t) describes the surface of the body as a function of time.

For a thin airfoil restricted to small oscillations, this may be written as

$$G = y - H(x,t) \qquad \text{(III-7)}$$

where

$H(x,t)$ is the function describing the position of the airfoil.

$H(x,t)$ can be written for harmonic pitch oscillations as

$$H(x,t) = R.P.[\alpha_o (x-x_o)\, e^{i\nu t}] \qquad \text{(III-8)}$$

where the time-varying angle of attack $\alpha(t)$ is given by

$$\alpha(t) = R.P.[\alpha_o e^{i\nu t}]$$

and α_o = maximum amplitude of pitch oscillation

x_o is the pitch axis

ν is the angular frequency of oscillation

$i = \sqrt{-1}$

R.P. = "real part of"

Inserting (III-8) into the flow tangency condition (III-6) gives, after linearization,

$$\phi_y(x,0) = v(x,0) = \alpha_o [U_o + i\nu(x - x_o)]\, e^{i\nu t} \qquad \text{(III-9)}$$

$$\text{on } y = 0$$

This is a condition for the normal velocity to be prescribed at the airfoil's mean position $y = 0$.

C. NONDIMENSIONALIZATION

The terms in equations (III-5) and (III-9) are dimensional. For the following calculations it is convenient to use non-dimensional quantities. Define non-dimensional time and length to be

$$\bar{x} = \frac{x}{c}$$

$$\bar{y} = \frac{y}{c} \qquad \text{(III-10)}$$

$$\bar{t} = \frac{tU_o}{c}$$

where

U_o = uniform velocity from infinity

c = reference length (blade semichord).

The velocity potential in equation (III-5) may be non-dimensionalized as follows. Let

$$\bar{\phi} = \frac{\phi}{U_o c}$$

Hence:

$$\phi = U_o c\bar{\phi}$$

$$\phi_x = U_o c\bar{\phi}_{\bar{x}}\left(\frac{1}{c}\right)$$

$$= U_o\bar{\phi}_{\bar{x}} \qquad \text{(III-11)}$$

and similarly for the other derivatives in (III-5), yielding

$$[(M^2-1) + (\gamma+1)M^2\bar{\phi}_{\bar{x}}]\bar{\phi}_{\bar{x}\bar{x}} - \bar{\phi}_{\bar{y}\bar{y}} + 2M^2\bar{\phi}_{\bar{x}\bar{t}} + M^2\bar{\phi}_{\bar{t}\bar{t}} = 0 \quad \text{(III-12)}$$

This equation is non-dimensional.

The boundary condition given in equation (III-9) may be non-dimensionalized in a similar fashion

$$\phi_y(x,0) = \alpha_o[U_o + i\nu(x - x_o)]\, e^{i\nu t} \quad \text{(III-9)}$$

Thus

$$U_o c\bar{\phi}_y = \alpha_o[U_o + ik\frac{U_o}{c}(c\bar{x} - c\bar{x}_o]\, e^{i\nu\bar{t}\frac{c}{U_o}} \quad \text{(III-13)}$$

where

$$k = \frac{\nu c}{U_o} = \text{Strouhal number or reduced frequency}$$

$$U_o c\bar{\phi}_{\bar{y}}\cdot\frac{1}{c} = \alpha_o U_o[1 + ik(\bar{x} - \bar{x}_o)]\, e^{ik\bar{t}} \quad \text{(III-14)}$$

Thus

$$\bar{\phi}_{\bar{y}} = \bar{v}(\bar{x},0) = \alpha_o[1 + ik(\bar{x} - \bar{x}_o)]\, e^{ik\bar{t}} \quad \text{(III-15)}$$

Because the final operations are linear in α_o, set $\alpha_o = 1$, yielding

$$\bar{\phi}_{\bar{y}} = \bar{v}(\bar{x},0) = [1 + ik(\bar{x} - \bar{x}_o)]\, e^{ik\bar{t}} \quad \text{(III-16)}$$

The overbars denoting nondimensional quantities will be dropped from the remainder of the paper. All further quantities shall be assumed appropriately non-dimensional. This yields the following final equations

$$[(M^2-1)+(\gamma+1)M^2\phi_x]\phi_{xx} - \phi_{yy} + 2M^2\phi_{xt} + M^2\phi_{tt} = 0 \tag{III-17}$$

and

$$\phi_y(x,0) = v(x,0) = [1 + ik(x - x_o)]\, e^{ikt} \tag{III-18}$$

where

all quantities are nondimensional and

$\alpha_o = 1$

D. HARMONIC OSCILLATIONS

In the case of harmonic oscillations, equation (III-17) may be simplified still further.

Let

$$\phi = \tilde{\phi} + \text{R.P.}[\psi e^{ikt}]$$

where

$\tilde{\phi}$ = non-dimensional steady flow potential

ψ = non-dimensional oscillatory flow potential

R.P. = "real part of"

Equation (III-17) then becomes

$$(1-M^2)\psi_{xx} + \psi_{yy} - M^2(\gamma+1)[\psi_x\psi_{xx} + \tilde{\phi}_x\psi_{xx} + \tilde{\phi}_{xx}\psi_x]$$

$$+ M^2k^2\psi - 2iMk^2\psi_x = 0 \qquad \text{(III-19)}$$

For M close to 1, this is a nonlinear mixed elliptic-hyperbolic partial differential equation with variable coefficients, the exact type depending on $\tilde{\phi}_x$ and $\tilde{\phi}_{xx}$. However, because flutter analysis is primarily concerned with the stability of small perturbations about a steady flow, the oscillatory component may be assumed small compared to the steady flow potential and therefore the product term $\psi_x\psi_{xx}$ may be neglected, yielding,

$$(1-M^2)\psi_{xx} + \psi_{yy} - M^2(\gamma+1)[\tilde{\phi}_x\psi_{xx} + \tilde{\phi}_{xx}\psi_x]$$

$$= 2iM^2k\psi_x + M^2k^2\psi = 0 \qquad \text{(III-20)}$$

IV. LINEARIZATION OF THE GOVERNING EQUATION

The basic flutter equation, (III-20), is still a non-linear, mixed elliptic-hyperbolic partial differential equation with variable coefficients and difficult to solve. It may yet be further simplified.

A. BASIC SOLUTION

For M = 1, equation (III-20) becomes

$$\psi_{yy} - (\gamma+1)[\tilde{\phi}_x\psi_{xx} + \tilde{\phi}_{xx}\psi_x] - 2ik\psi_x + k^2\psi = 0 \qquad \text{(IV-1)}$$

Now assume

$$\tilde{\phi}_x \approx w = \text{constant} \qquad \text{(IV-2)}$$

$$\tilde{\phi}_{xx} \approx 0$$

throughout the interblade channel. Setting

$$\tilde{\phi}_x(\gamma+1) = w(\gamma+1) = m^2 \qquad \text{(IV-3)}$$

yields

$$m^2\psi_{xx} - \psi_{yy} + 2ik\psi_x - k^2\psi = 0 \qquad \text{(IV-4)}$$

The solution to this equation is found in Garrick and Rubinow [5]

$$\psi(x,y) = -\frac{1}{m} \int_{x_\ell}^{x-my} v(s) J_o[\omega\sqrt{(x-s)^2-(my)^2}]\, e^{i\lambda(s-x)}\, ds \qquad \text{(IV-5a)}$$

for $y > 0$

and

$$\psi(x,y) = \frac{1}{m} \int_{x_\ell}^{x+my} v(s) J_o[\omega\sqrt{(x-s)^2-(my)^2}]\, e^{i\lambda(s-x)}\, dx \qquad \text{(IV-5b)}$$

for $y < 0$

where

$$v(x) = \lim_{y\to 0} \frac{\partial}{\partial y} \psi(x,y) .$$

v(x) may be obtained directly from the tangential flow boundary condition, and

$$\omega = \frac{k^2(1-m^2)}{m^4}$$

$$\lambda = \frac{k}{m^2}\sqrt{1+m^2} \approx \frac{k}{m^2}$$ (where this paper employs the approximation used by Gorelov [4])

x_ℓ = blade leading edge,

Gorelov [4], has proposed a further simplification. Set

$$z = my \tag{IV-6a}$$

$$\Psi(x,z) = \psi(x,y)\, e^{i\lambda x}\,. \tag{IV-6b}$$

Equation (IV-4) then becomes

$$\Psi_{xx} - \Psi_{zz} + \omega^2 \Psi = 0$$

with solution

$$\Psi(x,z) = -\frac{1}{m} \int_{x_\ell}^{x-z} v(s) J_o[\omega\sqrt{(x-s)^2 - z^2}]\, e^{i\lambda s} ds \tag{IV-7a}$$

$$z > 0$$

$$\Psi(x,z) = \frac{1}{m} \int_{x_\ell}^{x-z} v(s) J_o[\omega\sqrt{(x-s)^2 - z^2}]\, e^{i\lambda s} ds \tag{IV-7b}$$

$$z < 0$$

where

$$v(x) = me^{-i\lambda x} \lim_{z\to 0} \Psi_z(x,z)$$

$v(x)$ is obtained from the tangential flow boundary condition.

For a thin body immersed in the flow, the solutions for $y > 0$, $z > 0$, and $y < 0$, $z < 0$ apply above the body along left-running Mach lines, or below along right running Mach lines respectively.

B. BOUNDARY CONDITIONS

1. Flow Tangency Condition

The boundary condition comes from the tangential flow condition, (III-18)

$$v(x) = [1 + ik(x - x_o)] \qquad \text{(IV-8)}$$

2. Upstream Condition

The final linearlized equation is a hyperbolic differential equation with boundary condition

$$\psi(x,y) = 0 \qquad \text{(IV-9)}$$

when

$$x - x_\ell < |my|$$

for the solution shown in equations (IV-5) or

$$\Psi(x,z) = 0$$

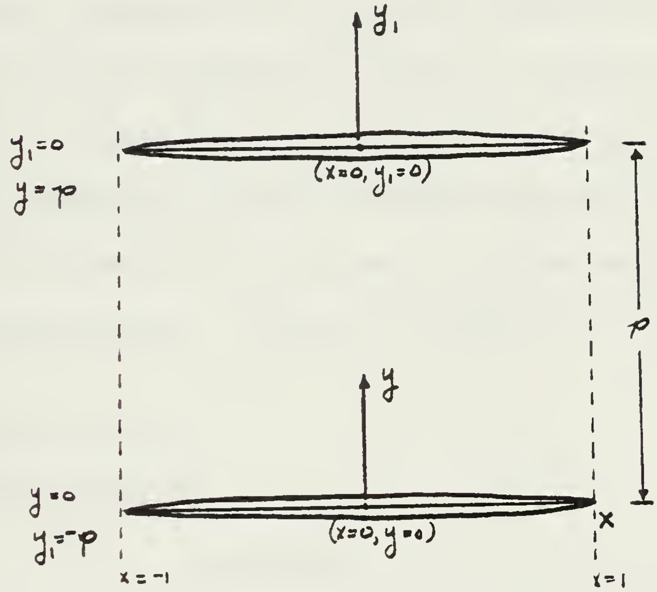
when

$$x - x_{\ell} < |z|$$

for the solution shown in equations (IV-7).

V. PROBLEM FORMULATION

A. CO-ORDINATE SYSTEM



Assume the geometry shown above. Both blades are thin airfoils of semichord c. All measurements are non-dimensional, normalized to c. The (x,y) co-ordinate system has its origin at the center of the reference (lower) blade. The (x,y_1) system is centered at the middle of the adjacent (upper) blade. The origin of the (x,y_1) system is located at (0,p) in the reference system. Generalizing this convention, the same symbols shall be used for the same quantities on both blades. Where discrimination is required, the quantity associated with the adjacent blade will be marked with

superscript ^{1}, the quantity associated with the reference blade will be either unsuperscripted or marked with a superscript $^{\circ}$.

Each blade is assumed to perform a small amplitude harmonic oscillation about its mid-chord point. Both blades are assumed to have identical reduced frequencies, k, and the motion of the adjacent blade lags that of the reference blade by a phase angle σ.

The blades are immersed in a uniform flow from the left at $M = 1$. The objective is to determine the oscillatory pressure distributions and aerodynamic forces generated by the blades' oscillations. Cascade solidity, $\tau = 2/p$.

B. BOUNDARY CONDITIONS

1. Upstream Condition

$$\psi = 0 \quad \text{whenever}$$

$$x + 1 < |my| \qquad \text{(V-1)}$$

and

$$x + 1 < |my_1|, \text{ simultaneously}$$

2. Flow Tangency Condition

Along the reference blade

$$\lim_{y \to 0} \psi_y(x,y) = (1 + ikx) \qquad \text{(V-2a)}$$

Along the adjacent blade

$$\lim_{y_1 \to 0} \psi_{y_1}(x,y_1) = (1 + ikx)e^{i\sigma} \qquad \text{(V-2b)}$$

where

σ is the phase angle between the blades oscillations

C. BASIC SOLUTION TECHNIQUE

Assume that the unsteady potential, ψ, may be written as the sum of four components

$$\psi(x,y) = \phi^{o}(x,y) + \psi^{o}(x,y) + \phi^{1}(x,y_1) + \psi^{1}(x,y_1) \qquad \text{(V-3)}$$

where:

ϕ^{o} = potential due to the reference blade alone, known from equation (IV-7)

ϕ^{1} = potential due to the adjacent blade alone, known from equation (IV-7)

ψ^{o} = interference potential required to satisfy tangential flow condition along reference blade, unknown

ψ^{1} = interference potential required to satisfy tangential flow condition along adjacent blade, unknown.

This total potential must satisfy the tangential flow condition at the plane of both the reference and adjacent blades. Thus

$$\phi^{o}_{y}(x,y=0) + \phi^{1}_{y_1}(x,y_1=-p) + \psi^{o}_{y}(x,y=0) + \psi^{1}_{y_1}(x,y_1=-p) \qquad \text{(V-4a)}$$

$$= (1 + ikx)$$

at the reference blade, and

$$\phi^{o}(x,y=p) + \phi^{1}_{y_1}(x,y_1=0) + \psi^{o}_{y}(x,y=p) + \psi^{1}_{y_1}(x,y_1=0)$$

$$= (1 + ikx)\, e^{i\sigma} \qquad \text{(V-4b)}$$

at the adjacent blade.

But from the unsteady potential solution for a single oscillating blade one has

$$\phi^{o}_{y}(x,y=0) = 1 + ikx \qquad \text{(V-5a)}$$

and

$$\phi^{1}_{y_1}(x,y_1=0) = (1 + ikx)\, e^{i\sigma} \qquad \text{(V-5b)}$$

Thus

$$\phi^{1}_{y_1}(x,y_1=-p) + \psi^{1}_{y_1}(x,y_1=-p) + \psi^{o}_{y}(x,y=0) = 0 \qquad \text{(V-6a)}$$

along the reference blade, and

$$\phi^{o}_{y}(x,y=p) + \psi^{o}_{y}(x,y=p) + \psi^{1}_{y_1}(x,y_1=0) = 0 \qquad \text{(V-6b)}$$

along the adjacent blade.

From equation (IV-7)

$$\phi^{o}(x,y) = -\frac{1}{m}\int_{-1}^{x-my} v^{o}(s)J_{o}[\omega\sqrt{(x-s)^{2}-(my)^{2}}]e^{i\lambda(s-x)}ds$$

$$y > 0 \qquad \text{(V-7a)}$$

$$= \frac{1}{m}\int_{-1}^{x+my} v^{o}(s)J_{o}[\omega\sqrt{(x-s)^{2}-(my)^{2}}]e^{i\lambda(s-x)}ds$$

$$y < 0 \qquad \text{(V-7b)}$$

where

$v^{o}(s) = 1 + iks$

$\lambda = k/m^{2}$

$m = (\gamma+1)w$

$\omega = \dfrac{k^{2}(1-m^{2})}{m^{4}}$

w = mean value of $\tilde{\phi}_x$ in the channel

$$\phi^{1}(x,y_1) = -\frac{1}{m}\int_{-1}^{x-my_1} v^{1}(s)J_{o}[\omega\sqrt{(x-s)^{2}-(my_1)^{2}}]e^{i\lambda(s-x)}ds$$

$$y_1 > 0 \qquad \text{(IV-8a)}$$

$$= \frac{1}{m}\int_{-1}^{x+my_1} v^{1}(s)J_{o}[\omega\sqrt{(x-s)^{2}-(my_1)^{2}}]e^{i\lambda(s-x)}ds$$

$$y_1 < 0 \qquad \text{(IV-8b)}$$

where

$$v^1(s) = (1 + iks)e^{i\sigma}$$

Henceforth attention will be restricted to the flow within the channel, $0 \leq y \leq p$, $-p \leq y_1 \leq 0$ leaving (IV-7a) and (IV-8b) as the governing equations of interest.

The two interference potentials are assumed to have forms identical to (IV-7a) and (IV-8b).

Set

$$\psi^o(x,y) = -\frac{1}{m}\int_{-1}^{x-my} u^o(s)J_o[\omega\sqrt{(x-s)^2-(my)^2}]e^{i\lambda(s-x)}ds \qquad y > 0 \tag{V-9a}$$

$$\psi^1(x,y_1) = \frac{1}{m}\int_{-1}^{x+m} u^1(s)J_o[\omega\sqrt{(x-s)^2-(my_1)^2}]e^{i\lambda(s-x)}ds \qquad y_1 < 0 \tag{V-9b}$$

where

$u^o(s)$ and $u^1(s)$ are unknown functions to be determined so as to satisfy equations (V-6)

Substitution of (V -7a), (V -8b) and (V -9) into (V -6) yields

$$u^o(x) + \psi^1_{y_1}(x,y_1{=}-p) + \phi^o_{y_1}(x,y_1{=}-p) = 0 \tag{V-10a}$$

$$u^1(x) + \psi_y^o(x,y=p) + \phi_y^o(x,y=p) = 0 \qquad \text{(V-10b)}$$

Recalling Gorelov's transformation discussed in [4] and shown in equations (IV-6) above, set

$$\Phi^o(x,z) = \phi^o(x,y) \quad e^{i\lambda x}$$

$$\Phi^1(x,z_1) = \phi^1(x,y_1) \quad e^{i\lambda x}$$

$$\Psi^o(x,z) = \psi^o(x,y) \quad e^{i\lambda x}$$

$$\Psi^1(x,z_1) = \psi^1(x,y_1) \quad e^{i\lambda x}$$

where $z = my$

$z_1 = my_1$

Then

$$\frac{e^{i\lambda x}}{m} u^o(x) + \Phi^1_{z_1}(x,z_1=-x_*) + \Psi^1_{z_1}(x,z_1=-x_*) = 0 \qquad \text{(V-11a)}$$

and

$$\frac{e^{i\lambda x}}{m} u^1(x) + \Phi^o_z(x,z = x_*) + \Psi^o_z(x,z=x_*) = 0 \qquad \text{(V-11b)}$$

where

$$x_* = mp .$$

To employ the collocation method, assume that $u^1(x)$ and $u^o(x)$ can be approximated as the sum of a set of elementary functions f_j so that

$$u^o(x) \simeq \sum_{j=0}^{n} \theta_j^o f_j(x) \tag{V-12a}$$

$$u^1(x) \simeq \sum_{j=0}^{n} \theta_j^1 f_j(x) \tag{V-12b}$$

where $f_j(x) = 0$ when $x \leq x_*-1$

Note that here both u^o and u^1 are expressed in terms of the same elementary functions, f_j.

Because of the slightly supersonic nature of the problem observe that $u^o(x) = 0$ and $u^1(x) = 0$ when $x \leq x_* -1$.

Equations (V-12) may now be rewritten as

$$e^{i\lambda x}\sum\theta_j^o f_j(x) + \frac{\partial}{\partial z_1}\int_{x_*-1}^{x+z_1} \sum\theta_j^1 f_j(s) J_o[\omega\sqrt{(x-s)^2-z_1^2}]e^{i\lambda s}ds$$

$$= -e^{i\sigma}\frac{\partial}{\partial z_1}\int_{-1}^{x+z_1} (1+iks)J_o[\omega\sqrt{(x-s)^2-z_1^2}]e^{i\lambda s}ds$$

$$\text{at} \quad z_1 = -x_* \tag{V-13a}$$

$$e^{i\lambda x}\sum\theta_j^1 f_j(x) - \frac{\partial}{\partial z}\int_{x_*-1}^{x-z} \sum\theta_j^o f_j(s) J_o[\omega\sqrt{(x-s)^2-z}] e^{i\lambda s} ds$$

$$= \frac{\partial}{\partial z}\int_{-1}^{x+z} (1+iks) J_o[\omega\sqrt{(x-s)^2 - z^2}] e^{i\lambda s} ds$$

$$\text{at} \quad z = x_* \qquad \text{(V-13b)}$$

where

$$f_j(x) = 0 \quad \text{for} \quad x \leq x_* - 1$$

This simplifies to

$$e^{i\lambda x}\sum\theta_j^o f_j(x) + \sum\theta_j^1 \{\frac{\partial}{\partial z_1}\int_{x_*-1}^{x+z_1} f_j(s) J_o[\omega\sqrt{(x-s)^2-z_1^2}] e^{i\lambda s} ds\}$$

$$= -e^{i\sigma}\frac{\partial}{\partial z_1}\int_{-1}^{x+z_1} (1+iks) J_o[\omega\sqrt{(x-s)^2-z_1^2}] e^{i\lambda s} ds$$

$$\text{at} \quad z_1 = -x_* \qquad \text{(V-14a)}$$

and

$$e^{i\lambda x}\sum\theta_j^1 f_j(x) - \sum\theta_j^o \{\frac{\partial}{\partial z}\int_{x_*-1}^{x-z} f_j(s) J_o[\omega\sqrt{(x-s)^2-z^2}] e^{i\lambda s} ds$$

$$= \frac{\partial}{\partial z}\int_{-1}^{x-z} (1+iks) J_o[\omega\sqrt{(x-s)^2-z^2}] e^{i\lambda s} ds \qquad \text{(V-14b)}$$

at $z = x_*$ where

$$f_j(x) = 0 \text{ for } x \le x_*-1.$$

Performing the indicated differentiation yields

$$e^{i\lambda x}\sum \theta_j^{o} f_j(x) + \sum \theta_j^{1}\{ \int_{x_*-1}^{x-x_*} f_j(s) \frac{J_1[\omega\sqrt{(x-s)^2-x_*^2}]\,\omega x_* e^{i\lambda s} ds}{\sqrt{(x-s)^2-x_*^2}}$$

$$+ f_j(x-x_*)e^{i\lambda(x-x_*)}\}$$

$$= e^{i\sigma}\{- \int_{-1}^{x-x_*} (1+iks) \frac{\omega x_* J_1[\omega\sqrt{(x-s)^2-x_*^2}]e^{i\lambda s} ds}{\sqrt{(x-s)^2-x_*^2}}$$

$$- [1+ik(x-x_*)]e^{i\lambda(x-x_*)}\} \qquad \text{(V-15a)}$$

and

$$e^{i\lambda x}\sum \theta_j^{1} f_j(x) + \sum \theta_j^{o}\{ \int_{x_*-1}^{x-x_*} f_j(s) \frac{J_1[\omega\sqrt{(x-s)^2-x_*^2}]\,\omega x_* e^{i\lambda s} ds}{\sqrt{(x-s)^2-x_*^2}}$$

$$+ f_j(x-x_*)e^{i\lambda(x-x_*)}\}$$

$$= - \int_{-1}^{x-x_*} (1+iks)\frac{\omega x_* J_1[\omega\sqrt{(x-s)^2-x_*^2}e^{i\lambda s} ds}{\sqrt{(x-s)^2-x_*^2}} - [1+ik(x-x_*)]e^{i\lambda(x-x_*)}$$

$$\text{(V-15b)}$$

Gorelov's formulation, equations [2.6, 2.7, 2.8, and 2.9] of [4], can be obtained directly from equations (V-15) by substituting

$$f_j(x) = \cos j\eta - \cos j\eta_*$$

$$v^o = \sum_{j=0}^{n} \mu_j^o \cos j\eta$$

$$v^1 = \sum_{j=0}^{n} \mu_j^1 \cos j\eta$$

where:

$$\eta = \cos(-x)$$

$$\eta_* = \cos(1-x_*)$$

In comparing the two systems care must be taken to note the differing symbols and coordinate systems. The corresponding quantities are:

Here	in[4]
$\theta_j^o,\ \theta_j^1$	$\nu_{o\sigma},\ \nu_{i\sigma}$
$\mu_j^o,\ \mu_j^1$	$\theta_{o\sigma},\ \theta_{i\sigma}$
$\eta = \cos^{-1}(-x)$	$\eta = \cos^{-1}(1-x)$
$\eta_* = \cos^{-1}(1-x_*)$	$\eta_* = \cos^{-1}(1-x_*)$
$z,\ z_1$	$y,\ y_1$
j	σ
σ	ψ

Here $-1 \le x \le 1$; in [4] $0 \le x \le 2$. This transformation accounts for the differing definitions of η. Making the substitutions results in the system

$$\sum_{j=0}^{n} \{\theta_j^o[\cos j\eta - (1-\delta_{o1}) \cos j\eta_*]$$

$$+ \theta_j^1 \int_{x_*-1}^{x-x_*} \frac{\partial}{\partial z_1} J_o[\omega\sqrt{(x-s)^2-z_1^2}][\cos j\hat{\eta} - (1-\delta_{io})\cos j\eta_*]e^{i\lambda s}ds$$

$$+ \theta_i^1[\cos j\bar{\eta} - (1-\delta_{io})\cos j\eta_*]e^{i\lambda(x-x_*)}\}$$

$$= - \sum_{j=0}^{n}\{-\mu_j^1 \int_{-1}^{x-x_*} \frac{\partial}{\partial z_1} J_o[\sqrt{(x-s)^2 - z_1^2}] \cos j\hat{\eta} e^{i\lambda s}ds$$

$$- \mu_j^1 \cos j\bar{\eta} e^{i\lambda(x-x_*)}\} \qquad \text{(V-16a)}$$

$$\text{at } z_1 = -x_*$$

$$x > x_*-1$$

and

$$\sum\{\theta_j^1[\cos j\eta - (1-\delta_{oj}) \cos j\eta_*]$$

$$+ \theta_j^o \int_{x_*-1}^{x-x_*} \frac{\partial}{\partial z} J_o[\omega\sqrt{(x-s)^2-z^2}][\cos j\eta-(1-\delta_{oj})\cos j\eta_*]e^{i\lambda s}ds$$

$$+ \theta_j^o[\cos j\bar{\eta} - (1-\delta_{oj})\cos j\eta_*]e^{i\lambda(x-x_*)}\}$$

$$= \sum_{j=0}^{n} \{\mu_j^o \int_{-1}^{x-x_*} \frac{\partial}{\partial z} J_o[\omega\sqrt{(x-s)^2-z^2}]\cos j\hat{\eta}e^{i\lambda s}ds$$

$$- \mu_j^o \cos j\bar{\eta}e^{i\lambda(x-x_*)}\} \qquad \text{(V-16b)}$$

where:

$$\eta = \text{arc cos } (-x)$$

$$\eta_* = \text{arc cos } (1-x_*)$$

$$\bar{\eta} = \text{arc cos } (-s)$$

$$\hat{\eta} = \text{arc cos } (-x+x_*)$$

$$\delta_{oj} = \text{Dirac } \delta \text{ function } = \begin{cases} 1 & \text{for } j = 0 \\ 0 & \text{for } j \neq 0 \end{cases}$$

Given the change in coordinates and notation, this system is equivalent to that shown in [4].

This was the system programmed for computer solution. Because the function, $f_j(x)$, is unaffected by the differentiation with respect to y (or z) the exact form used need not be specified, so that the system shown in (V-15) may be programmed with f undetermined. A subroutine may be written to return the function desired and the remaining program left perfectly general. In the program developed with this thesis both the Gorelov functions shown above and the Legendre polynomials were employed. All the integrals may now be evaluated at n+1 points, x_i, on both blades in $mp-1 < x_i < 1$ and the resulting linear system solved for θ_j^o and θ_j^1, j=1,2,...,n+1.

The interference potentials may be constructed by taking

$$\psi^o(x,z) = \frac{-1}{m} \int_1^{x-z} [\sum \theta_j^o f_j(x)] J_o[\omega\sqrt{(x-s)^2-z^2}] e^{i\lambda s} ds \qquad z > 0 \qquad \text{(V-17a)}$$

$$\psi^1(x,z_1) = \frac{1}{m} \int_{-1}^{x+z_1} [\sum \theta_j^1 f_j(x)] J_o[\omega\sqrt{(x-s)^2-z_1^2}] e^{i\lambda s} ds \qquad z_1 < 0 \qquad \text{(V-17b)}$$

where

$$f_j(x) = 0, \quad \text{for all} \quad x \leq x_*-1$$

Once the potentials have been calculated as outlined above, the surface pressure may be calculated using the relationship

$$C_p = -2(\psi_x + ik\psi) \tag{V-18a}$$

$$= -2[\Psi_x + i(k-\lambda)\Psi]\, e^{-i\lambda x} \tag{V-18b}$$

Because all the plates are assumed to be in steady oscillation with uniform phase shift, σ, between neighboring plates, then

$$v^1(x) = v^o(x)e^{i\sigma} , \quad u^1(x) = u^o(x)e^{i\sigma} , \quad \Psi(x,y) = -\Psi(x,-y)$$

in this case

$$C_{\ell_\alpha} = 2 \int_{-1}^{1} [\Psi_x(x,+0) + i(k-\lambda)\Psi(x,+0)]\, e^{-i\lambda x} dx \tag{V-19}$$

where

$$\Psi(x_1,+0) = \frac{-1}{m} \int_{-1}^{x} [(1+iks)+u^o(s)]\, J_o[\omega(x-s)]\, e^{i\lambda(s)} ds$$

$$+ \frac{e^{i\sigma}}{m} \int_{-1}^{x-x_*} [(1+iks)+u^o(s)]\, J_o[\omega\sqrt{(x-s)^2-(x_*)^2}]\, e^{i\lambda(s)} ds$$

$$\text{where } x_* = mp. \tag{V-20}$$

Results from this approach, in the form of values of C_{ℓ_α} for $k = 0.1$, at various values of w are presented in the results for

approximations based both on Gorelov's formulation, and on the Legendre polynomials.

D. COLLOCATION SOLUTION OF THE POTENTIAL EQUATION EXPANDED FOR SMALL k,

In order to provide a partially independent check of the results of the main program, the Gorelov function representation of the collocation solution was expanded for small k, and solved at two collocation points, n = 2. The resulting potentials, and partial derivatives with respect to x and y were then used to replace the corresponding numerical routines in the main program. The output resulting from the approximations were compared with the purely numerical results obtained from the computer program.

1. Solution For The Unknown Potential Coefficients

The basic system of linear equations used to determine the unknown coefficients is

$$\frac{1}{m}e^{i\lambda x}u^{o} + \phi^{1}_{z_1} + \psi^{1}_{z_1} = 0 , \quad z = 0, \quad z_1 = -x_* = -mp \tag{V-21a}$$

$$\frac{1}{m}e^{i\lambda x}u^{1} + \phi^{o}_{z} + \psi^{o}_{z} = 0 , \quad z_1 = 0, \quad z = x_* = mp \tag{V-21b}$$

where

$$0 = u^{o}(x) = u^{1}(x) \quad \text{when} \quad x \leq -1+x_*$$

otherwise

$$u^o = \sum_{j=1}^{n} \theta_j^o(\cos j\eta - \cos j\eta_*) + \theta_o^o$$

$$u^1 = \sum_{j=1}^{n} \theta_j^1(\cos j\eta - \cos j\eta_*) + \theta_o^1$$

where

$$\eta = \text{arc cos } (-x)$$

$$\eta_* = \text{arc cos } (1 - x_*) \ .$$

Thus, for n = 2, the system becomes

$$e^{i\lambda x}\{\theta_o^o + \theta_1^o(\cos \eta - \cos \eta_*)\}$$

$$+ \frac{\partial}{\partial z_1} \int_{-1}^{x+z_1} v^1(s) J_o \left[\omega \sqrt{(x-s)^2 - z_1^2}\right] e^{i\lambda s} ds$$

$$+ \frac{\partial}{\partial z_1} \int_{x_*-1}^{x+z_1} u^1(s) J_o \left[\omega \sqrt{(x-s)^2 - z_1^2}\right] e^{i\lambda s} ds = 0$$

$$z_1 = -x_* \qquad \text{(V-22a)}$$

along the reference blade and

$$e^{i\lambda x}\{\theta_o^1 + \theta_1^1(\cos\eta - \cos\eta_*)\} - \frac{\partial}{\partial z}\int_{-1}^{x-z} v^o(s)J_o[\omega\sqrt{(x-s)^2-z^2}]e^{i\lambda s}ds$$

$$- \frac{\partial}{\partial z}\int_{x_*-1}^{x-z} u^o(s)J_o[\omega\sqrt{(x-s)^2 - z^2}]e^{i\lambda s}ds \Bigg|_{z = x_*} . \qquad \text{(V-22b)}$$

along the adjacent blade

where $u^o(s) = \theta_o^o + \theta_1^o(\cos\eta - \cos\eta_*)$

$$u^1(s) = \theta_o^1 + \theta_1^1(\cos\eta - \cos\eta_*)$$

$$v^o(s) = 1 + iks$$

$$v^1(s) = (1 + iks)e^{i\sigma}$$

For k sufficiently small, this system may be further simplified by the following approximations

$$J_o[\omega\ \overline{(x-s)^2-z^2}] \approx 1 - 0(\omega^2) \approx 1 \qquad \text{(V-23a)}$$

$$J_o[\omega\sqrt{(x-s)^2-z^2}] \approx 1 - 0(\omega^2) \approx 1 \qquad \text{(V-23b)}$$

$$e^{i\lambda x} \approx 1 + i\lambda x - 0(\lambda^2 x^2) \approx 1 + i\lambda x$$

$$e^{i\lambda s} \approx 1 + i\lambda s - 0(\lambda^2 s^2) \approx 1 + i\lambda s$$

where $0(\omega^2)$ means "of the order of magnitude of ω^2"

$-1 \leq x \leq 1$, $-1 \leq s \leq x-x_*$

$\lambda = k/m^2$, $\omega^2 = \dfrac{k^2(1-m^2)}{m^4}$

The interference source distributions may be replaced by

$$u^o(s) = \theta^o_o + \theta^o_1(-s + x_*-1)$$

$$u^1(s) = \theta^1_o + \theta^1_1(-s + x_*-1) \ .$$

If higher order terms are neglected, the result is a system linear in k and λ

$$(1+i\lambda x)[\theta^o_o + \theta^o_1(-x-1+x_*)] + \frac{\partial e^{i\sigma}}{\partial z_1} \int_{-1}^{x+z_1} (1+iks)(1+i\lambda s)ds$$

$$+ \frac{\partial}{\partial z_1} \int_{x_*-1}^{x+z_1} [\theta^1_o + \theta^1_1(-s-1+x_*)](1+i\lambda s)ds = 0$$

$$z_1 = -x_* \qquad \text{(V-24a)}$$

$$(1+i\lambda x)[\theta^1_o + \theta^1_1(-x-1+x_*)] - \frac{\partial}{\partial z} \int_{-1}^{x-z} (1-iks)(1+i\lambda s)ds$$

$$- \frac{\partial}{\partial z} \int_{x_*-1}^{x-z} [\theta^o_o + \theta^o_1(-s-1+x_*)](1+i\lambda s)ds = 0$$

$$z = x_* \qquad \text{(V-24b)}$$

Product terms containing $(k\lambda) = \frac{k^2}{m^2}$ may be neglected as of higher order in k, yielding

$$(1+i\lambda x)[\theta_o^o + \theta_1^o(-x-1+x_*)] + \frac{\partial e^{i\sigma}}{\partial z_1} \int_{-1}^{x+z_1} [1+i(\lambda+k)s]ds$$

$$+ \frac{\partial}{\partial z_1} \int_{x_*-1}^{x+z_1} \{[\theta_o^1 + \theta_1^1(-s-1+x_*)]$$

$$+ i\lambda s[\theta_o^1 + \theta_1^1(-s-1+x_*)]\}ds = 0$$

$$z_1 = -x_* \qquad \text{(V-25a)}$$

$$(1+i\lambda x)[\theta_o^1 + \theta_1^1(-x-1+x_*)] - \frac{\partial}{\partial z} \int_{-1}^{x-z} [1+i(\lambda+k)s]ds$$

$$- \frac{\partial}{\partial z} \int_{x_*-1}^{x-z} \{[\theta_o^o + \theta_1^o(-s-1+x_*)] + i\lambda s[\theta_1^o + \theta_1^o(-s-1+x_*)]\}ds = 0$$

$$z = x_* \qquad \text{(V-25b)}$$

Evaluating the indicated derivatives yields

$$(1+i\lambda x)[\theta_o^o + \theta_1^o(-x-1+x_*)] + [1+i(k+\lambda)(x-x_*)]e^{i\sigma}$$

$$+ [\theta_o^1 + \theta_1^1(2x_*-x-1)]+i\lambda(x-x_*)[\theta_o^1+\theta_1^1(2x_*-x-1)]\} = 0$$

(V-26a)

$$(1+i\lambda x)[\theta_o^1+\theta_1^1(-x-1+x_*)] + [1+i(\lambda+k)(x-x_*)]$$

$$+ \{[\theta_o^o+\theta_1^o(2x_*-x-1)] + i\lambda(x-x_*)[\theta_o^o+\theta_1^o(2x_*-x-1)]\} = 0$$

(V-26b)

Thus:

$$\theta_o^o(1+i\lambda x) + \theta_1^o[(-x-1+x_*) + i\lambda x(-x-1+x_*)]$$

$$+ \theta_o^1[1\ i\lambda(x-x_*)] + \theta_1^1[(2x_*-x-1) + i\lambda(x-x_*)(2x_*-x-1)]$$

$$= -e^{i\sigma}[1 + i(k+\lambda)(x-x_*)] \tag{V-27a}$$

$$\theta_o^1[1 + i\lambda x] + \theta_1^1[(-x+1-x_*) + i\lambda x(-x+1-x_*)]$$

$$+ \theta_o^o[1+i\lambda x(x-x_*)] + \theta_1^o[(2x_*-x-1)+i\lambda(x-x_*)(2x_*-x-1)]$$

$$= -[1 + i(\lambda+k)(x-x_*)] \tag{V-27b}$$

This system may be solved at two points, x_1 and x_2, for θ_o^o, θ_1^o, θ_o^1, and θ_1^1.

2. <u>Calculation Of The Potential</u>

The potential is given by

$$\psi(x,y) = \Psi(x,z)e^{-i\lambda x} \tag{V-28}$$

where

$$\Psi(x,z) = -\frac{1}{m}\int_{-1}^{x}[v^{o}(s)+u^{o}(s)]J_{o}[\omega\sqrt{(x-s)^{2}}]e^{i\lambda s}ds$$

$$+\frac{1}{m}\int_{-1}^{x-x_{*}}[v^{1}(s)+u^{1}(s)]J_{1}[\omega\sqrt{(x-s)^{2}-x_{*}^{2}}]e^{i\lambda s}ds.$$

$$u^{o}(s) = u^{1}(s) = 0 \quad \text{for all} \quad s \leq x_{*}-1 .$$

Thus

$$\Psi(x,z) = -\frac{1}{m}\int_{-1}^{x}v^{o}(s)J_{o}[\omega\sqrt{(x-s)^{2}}]e^{i\lambda s}ds$$

$$-\frac{1}{m}\int_{x_{*}-1}^{x}u^{o}(s)J_{o}[\omega\sqrt{(x-s)^{2}}]e^{i\lambda s}ds$$

$$+\frac{1}{m}\int_{-1}^{x-x_{*}}v^{1}(s)J_{o}[\omega\sqrt{(x-s)^{2}-x_{*}^{2}}]e^{i\lambda s}ds$$

$$+\frac{1}{m}\int_{x_{*}-1}^{x-x_{*}}u^{1}(s)J_{o}[\omega\sqrt{(x-s)^{2}-x_{*}^{2}}]e^{i\lambda s}ds \tag{V-29}$$

Making the same small frequency approximations as in the previous section yields

$$\Psi(x,z) = -\frac{1}{m}\int_{-1}^{x} v^{o}(s)(1+i\lambda s)\,ds$$

$$-\frac{1}{m}\int_{x_*-1}^{x} u^{o}(s)(1+i\lambda s)\,ds$$

$$+\frac{1}{m}\int_{-1}^{x-x_*} v^{1}(s)(1+i\lambda s)\,ds$$

$$+\frac{1}{m}\int_{x_*-1}^{x-x_*} u^{1}(s)(1+i\lambda s)\,ds \qquad \text{(V-30)}$$

From the general formulation

$$\Psi = \Phi^{1} + \Phi^{o} + \Psi^{1} + \Psi^{o} \qquad \text{(V-31)}$$

Thus, along the reference blade

$$-m\Phi^{o}(x,z=0) = \int_{-1}^{x} v^{o}(s)(1+i\lambda s)\,ds = \int_{-1}^{x} (1+iks)(1+i\lambda s)\,ds$$

$$= \int_{-1}^{x} [1+i(k+\lambda)s]\,ds = \left[s+i(k+\lambda)\frac{s^2}{2}\right]_{-1}^{x} \qquad \text{(V-32)}$$

$$= x + i\frac{(k+\lambda)}{2}x + 1 - i\left(\frac{k+\lambda}{2}\right)$$

$$\Phi^{o}(x,z=0) = -\frac{1}{m}[(1+x) + i(\frac{k+\lambda}{2})(x^2-1)] \qquad (V-33)$$

By inspection

$$\Phi^{1}(x,z_1=-x_*) = \frac{e^{i\sigma}}{m}\{(1+x-x_*) + i(\frac{k+\lambda}{2})[(x-x_*)^2-1]\} \qquad (V-34)$$

$$-m\Psi^{o}(x,z=0) = \int_{x_*-1}^{x} u^{o}(s)(1+i\lambda s)ds \qquad (V-35)$$

$$= \int_{x_*-1}^{x} [\theta_o^o + \theta_1^o(-s+x_*-1)](1+i\lambda s)ds$$

$$= \int_{x_*-1}^{x} \theta_o^o(1+i\lambda s) + \theta_1^o(-s+x_*-1)(1+i\lambda s)ds$$

$$= \theta_o^o[(x+1-x_*)+\frac{i\lambda}{2}(x^2-x_*^2+2x_*-1)]$$

$$+ \int_{x_*-1}^{x} \theta_1^o[-x+x_*-1+i\lambda(-s^2+sx_*-s)]ds$$

$$= \theta_o^o[(x+1-x_*)+\frac{i\lambda}{2}(x^2-x_*^2+2x_*-1)]$$

$$+ \theta_1^o\{(-\frac{s^2}{2}+sx_*-s)+i\lambda(-\frac{s^3}{3}+\frac{s^2x_*}{2}-\frac{s^2}{2})]_{x_*-1}^{x}\} \qquad (V-36)$$

$$= \theta_o^o[(x+1-x_*)+\frac{i\lambda}{2}(x^2-x_*^2+2x_*-1)]$$

$$+ \theta_1^o\{(-\frac{x^2}{2}+x_*-x)-[-\frac{(x_*-1)^2}{2}+x_*(x_*-1)-(x_*-1)]$$

$$+ i\lambda[(-\frac{x^3}{3}+\frac{x^2x_*}{2}-\frac{x^2}{2})+\frac{(x_*-1)^3}{3}-x_*\frac{(x_*-1)^2}{2}+\frac{(x_*-1)^2}{2}]\}$$

$$= \theta_o^o[(x+1-x_*)+\frac{i\lambda}{2}(x^2-x_*^2+2x_*-1)]$$

$$+ \theta_1^o \; -\frac{x^2}{2}+x(x_*-1)+\frac{(x_*-1)^2}{2} - (x_*-1)^2$$

$$+ i\lambda[-\frac{x^3}{3} + x^2\frac{(x_*-1)}{2} + \frac{x_*^3}{3} - x_*^2 + x_* - \frac{1}{3}$$

$$- \frac{x_*^3}{2} + x_*^2 - \frac{x_*}{2} + \frac{x_*^2}{2} - x_* + \frac{1}{2}]\}$$

$$= \theta_o^o[(x+1-x_*) + \frac{i\lambda}{2}(x^2 - x_*^2 + 2x_* - 1)]$$

$$+ \theta_1^o\{- \frac{x^2}{2} + x(x_*-1) - \frac{(x_*-1)^2}{2}$$

$$+ i\lambda[- \frac{x^3}{3} + \frac{x^2(x_*-1)}{2} - \frac{x_*^3}{6} + \frac{x_*^2}{2} - \frac{x_*}{2} + \frac{1}{6}]\}$$

$$\Psi^o = -\frac{1}{m}\{\theta^o_o[(x+1-x_*) + \frac{i\lambda}{2}(x^2 + x_*^2 + 2x_*-1)]$$

$$+ \theta^o_1\{-\frac{x^2}{2} + x(x_*-1) - \frac{(x_*-1)^2}{2}$$

$$+ i\lambda[-\frac{x^3}{3} + \frac{x^2(x_*-1)}{2} - \frac{(x_*-1)^3}{6}]\}\} \qquad \text{(V-37)}$$

$m\psi^1(x,z_1=-x_*)$ may be evaluted by substituting $x-x_*$ for x in the expression for $m\Psi^o$ and exchanging θ^1_o and θ^1_1 for θ^o_o and θ^o_1

$$m\Psi^1(x,z_1=-x_*) = \theta^1_o\{[(x-x_*)+1-x_*] + \frac{i\lambda}{2}[(x-x_*)^2 - x_*^2 + 2x_* - 1]\}$$

$$+ \theta^1_1\{\frac{-(x-x_*)^2}{2} + (x-x_*)(x_*-1) - \frac{(x_*-1)^2}{2}$$

$$+ i\lambda[-\frac{(x-x_*)^3}{3} + \frac{x^2(x_*-1)}{2} - \frac{(x_*-1)^3}{6}]\} \qquad \text{(V-38)}$$

$$= \theta^1_o\ (x+1-2x_*) + \frac{i\lambda}{2}[(x^2-2xx_*+x_2^2) - x_*^2 + 2x_* - 1]\}$$

$$+ \theta^1_1\{-\frac{(x^2-2xx_*+x_*^2)}{2} + (x - \frac{3}{2}x_* + \frac{1}{2})(x_*-1)$$

$$+ i\lambda[-\frac{x^3}{3} + x^2x_* - xx_*^2 + \frac{x_*^3}{3} + (x^2-2xx_*+x_*^2)\frac{(x_*-1)}{2}$$

$$- \frac{(x_*-1)^3}{6}]\}$$

$$= \theta_o^1\{(x+1-2x_*) + \frac{i\lambda}{2}[x^2 - 2xx_* + 2x_* - 1]$$

$$+ \theta_1^1\{-\frac{x^2}{2} + xx_* - \frac{x_*^2}{2} + x(x-1) + \frac{1}{2}(1-3x_*)(x_*-1)$$

$$+ i\lambda[-\frac{x^3}{3} + x^2\frac{(3x_*-1)}{2} + x(-\frac{x_*^2-2xx_*^2-1)}{2}$$

$$+ \frac{x_*^3}{3} + \frac{x_*^3}{2} - \frac{1}{2}]\}$$

$\psi^1(x,z_1=-x_*)$

$$= \frac{1}{m}\{\theta_o^1\{(x+1-2x_*) + \frac{i\lambda}{2}[x^2 - 2xx_* + 2x_* - 1]$$

$$+ \theta_1^1\{[-\frac{x^2}{2} + x(2x_*-1) - xx_*^2 + 2x_* - \frac{1}{2}]$$

$$+ i\lambda[-\frac{x^3}{3} + x^2\frac{(3x_*-1)}{2} + x\frac{(-3x_*^2-1)}{2} + 5\frac{x_*^3}{6} - \frac{1}{2}]\}\} \tag{V-39}$$

A comparison of the results for the full program and the approximation is given below for $k = 0.01$, $w = 0.05$, $\sigma = \rho$, $n = 2$, yielding $\omega^2 \simeq 6.1 \times 10^{-3}$, $\lambda \approx 0.083$

		Full program	Approx
$x = .1285$			
	ϕ	-5.363, .327i	-5.379, .549i
	ϕ_x	-8.634, .361i	-8.66, .379i
$x = .5643$			
	ϕ	-9.754, .809i	-9.804, 1.000i
	ϕ_x	-12.234, .692i	-12.272, .7608i
	C_{ℓ_α}	+31.658,-3.1032i	C_{ℓ_α} 31.7019,-3.2165i

VI. RESULTS

The collocation method was used to solve the partial differential equation resulting from the Gorelov approximation of transonic potential flow in an unstaggered cascade. The system was solved using both the spanning functions proposed by Gorelov in [4], resulting in the equations (V-16); and the Legendre polynomials, resulting in equations (V-15) with f_j replaced by the Legendre polynomial, P_j. The resulting values of C_{ℓ_α} for k = .1, τ = 1, σ = 1 and seven collocation points on each blade are presented in figures VI-2, VI-3, and VI-4.

Figure VI -1 presents a diagram which is useful in commenting on the other results. This shows the location of the collocation points and first three interference reflections as a function of w expressed as a percentage of that portion of the chord subject to reflection. The collocation points are equally spaced throughout this interval, 12.5% from the leading edge of the interference zone, 12.5% between each pair of points and 12.5% from the blade trailing edge. The independent variable, w, is plotted vertically so that the dependent variable, percent of chord subject to interference, may be more conveniently visualized along the blade. (The curves are not precisely linear, but are very nearly so in the range shown.)

Figure VI -2 shows the C_{ℓ_α} calculated with k = 0.0 in comparison with the results obtained from Ackeret theory.

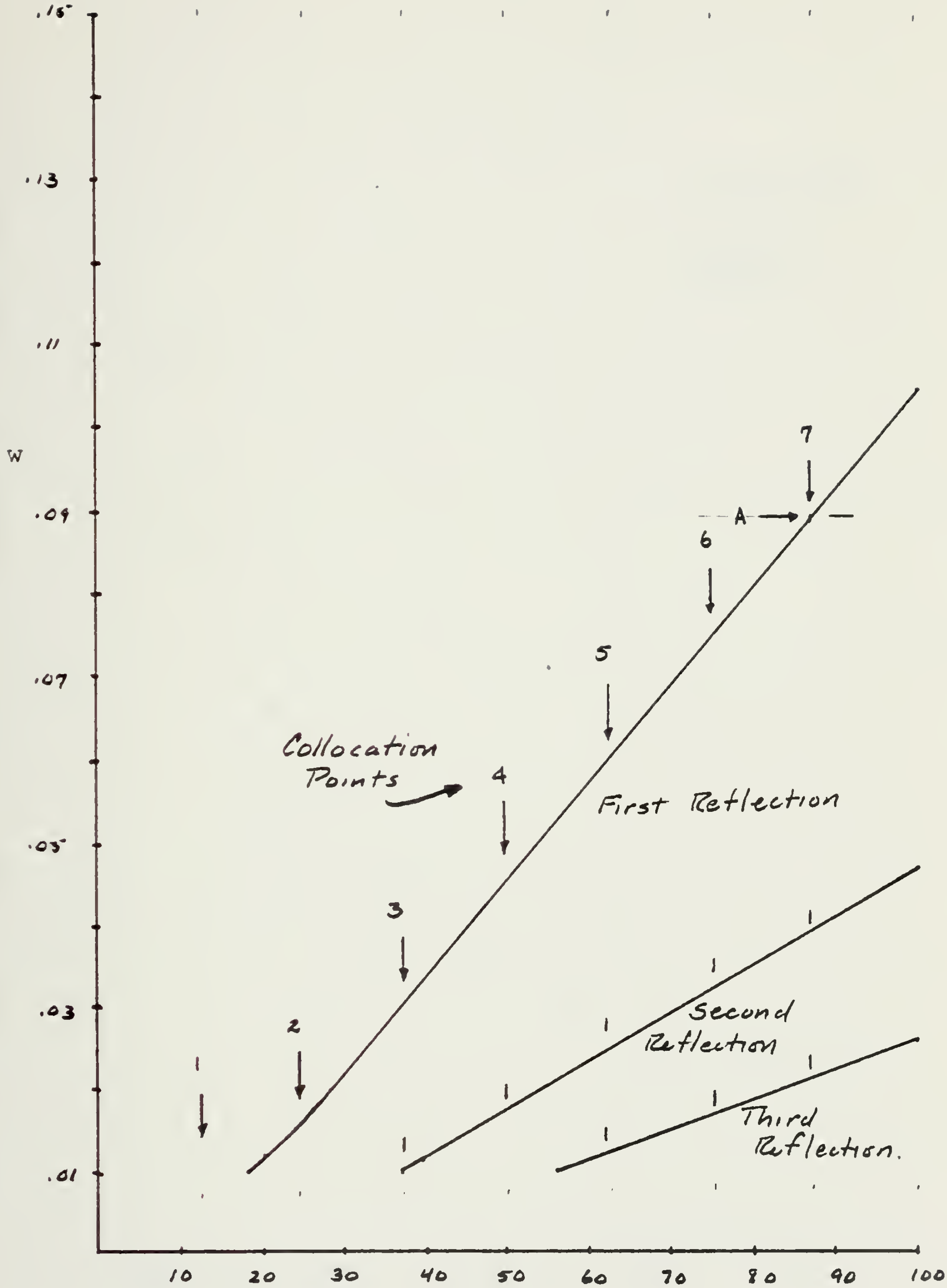
Agreement is good where there is no reflection and the portion of the blade subject to interference is affected by a constant interference potential, $w \geq 0.11$. Throughout the rest of the curve the results calculated here oscillate above and below the theoretical values. This appears to be due to the discrete nature of the approximation used in the collocation method. Rarely is the fraction of the chord subject to interference reflection equal to the fraction of the collocation points which feel it. Where the collocation point fraction lags, as near $w = 0.6$, the collocation results are lower than those due to Ackeret theory. When the collocation point fraction leads, as it does for $w \leq .04$ and briefly for $w \approx .08$, the the collocation results are higher than those due to Ackeret theory. The fault appears to be an intrinsic feature of the small number of points sampled. This results in a set of coefficients similar to those which would be obtained from a generalized Fourier series based on the integration of the Taylor series expansion about each point. This obviously cannot be a good approximation when both the function and its derivative are discontinuous at the reflections.

Figures VI -3 and VI -4 show the results of using Legendre polynomials and Gorelov's functions as spanning functions. The results for both formulations are identical. Gorelov's results are presented for comparison. Agreement is good for $w > 0.05$ except for an anomalous point, marked A .

It is believed that this anomaly is due to the location of the first reflection just ahead of the last collocation point (cf. "A" on Figure VI -1). This will yield a very small contribution from the reflection potential to the linear system from which the collocation points are determined. The resulting system will have a large dynamic range and may be ill-conditioned.

The discrepancy between these results, and those in [3] for $w < 0.5$ is still unexplained, as is the outlying value for $w = 0.5$.

The discontinuities in the imaginary results are believed to be due primarily to the reflection/collocation interaction explained above.



Percent of chord subject to interference

FIGURE VI-1. Location of Reflections and Collocation Points Shown as Percent of Chord Subject to Interference

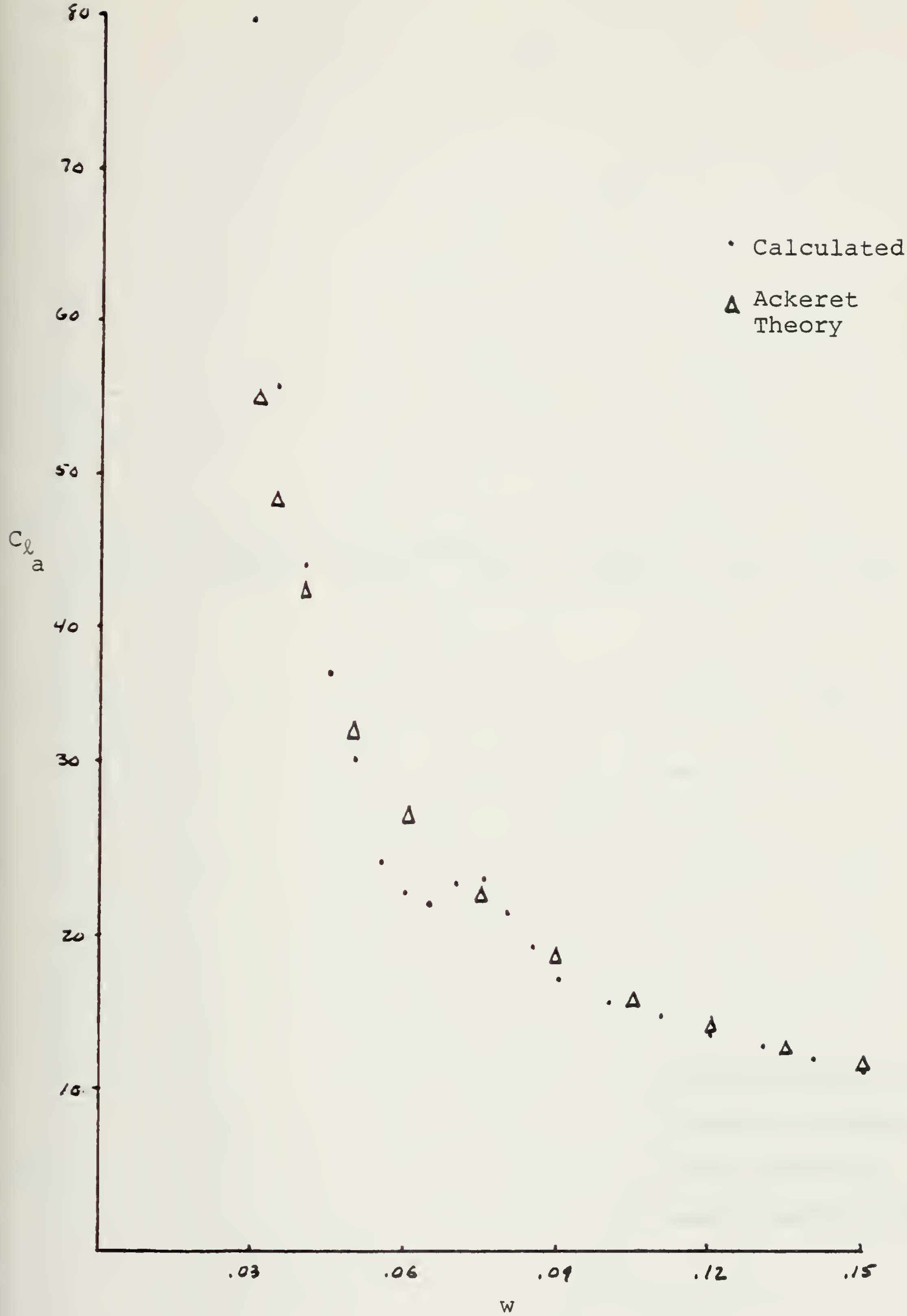


FIGURE VI-2. Comparison of $C\ell_{\alpha}$-vs-w to that Obtained from Ackeret Theory

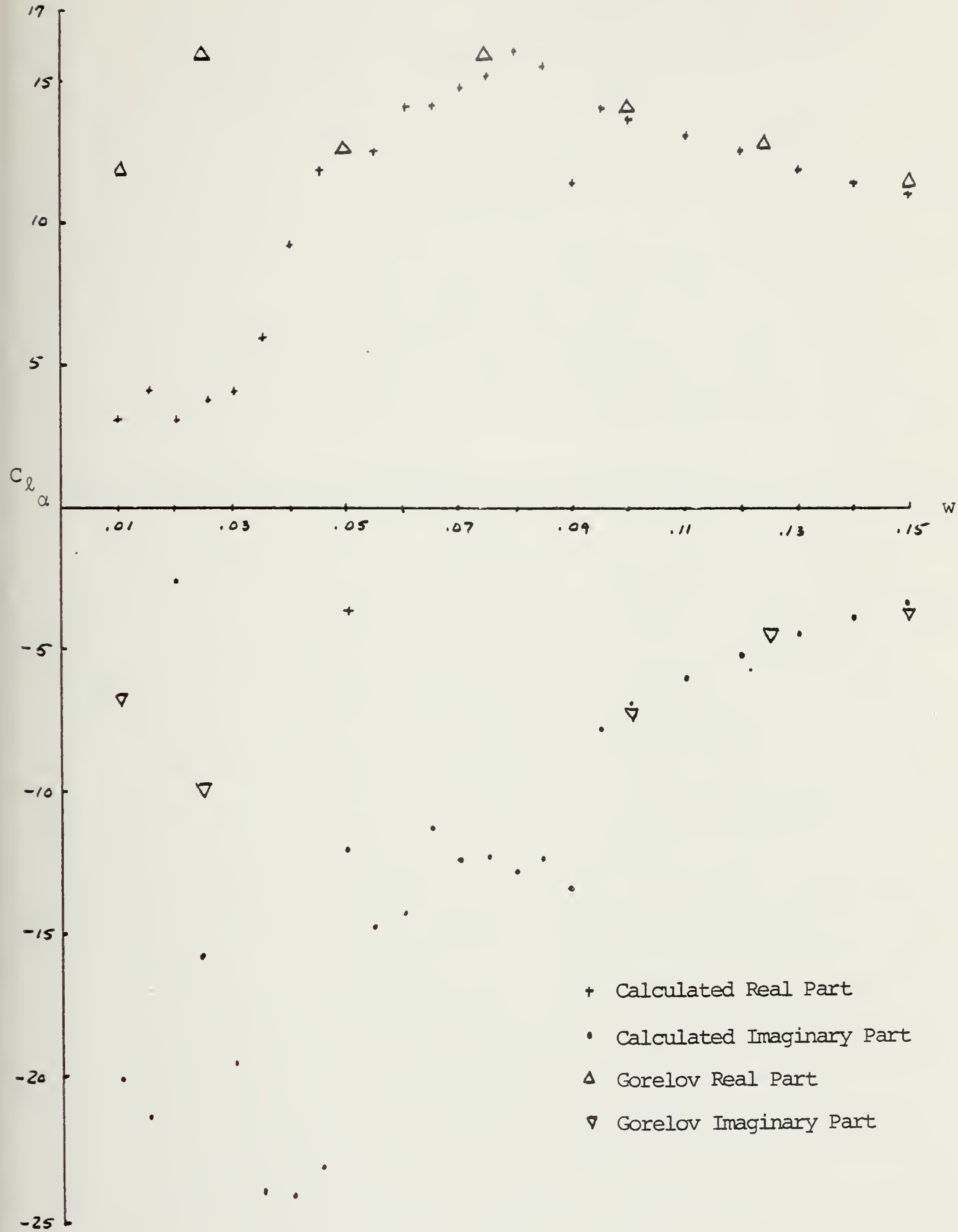


FIGURE VI-3. Plot of $C\ell\alpha$-vs-w, Legendre Polynomials
$k = 0.1$, $\tau = 1.0$, $\sigma = \pi$, $n = 7$
compared with Gorelov's results

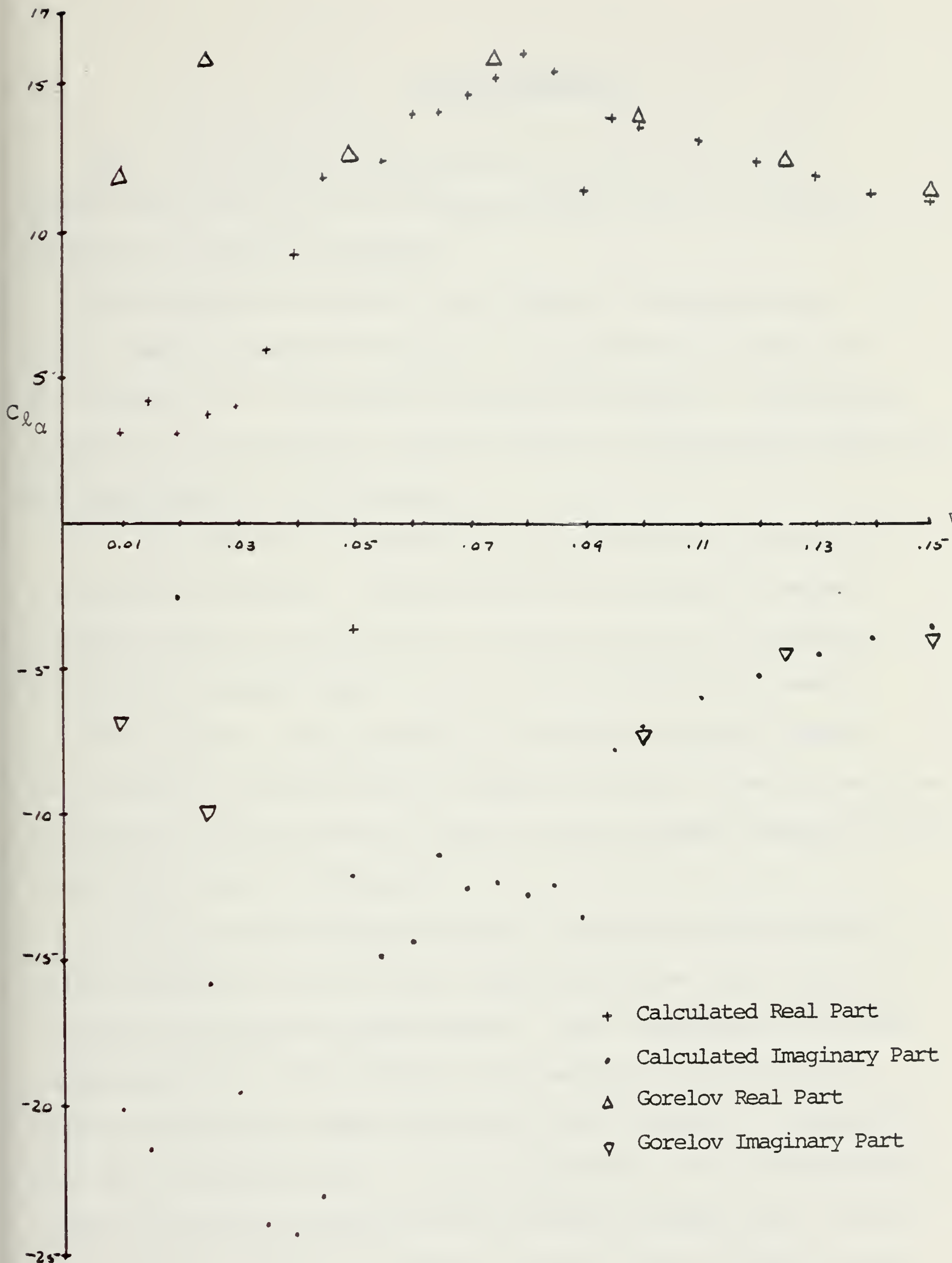


FIGURE VI-4. Plot of C_{ℓ_α}-vs-w, Gorelov Spanning Function
$k = 0.1$, $\tau = 1.0$, $\sigma = \pi$, $n = 7$
compared with Gorelov's results

VII. RECOMMENDATIONS

There are two recommendations to be made about the techniques used in the collocation method, and a new area in which it might be employed.

The program developed in the course of writing this thesis employs adaptive Simpson's integration to calculate the elements of a completely determined system. This system is solved to provide the coefficients of the spanning functions. Two improvements may be made:

1. The Simpson's integration scheme may be replaced by a Gaussian integrator. Experience has shown that several thousand function evaluations are required by the Simpson's integration routine when C_{ℓ_α} is to be evaluated for small w. This entails large amounts of computer time and leads to increased accumulations of numerical error. Use of Gaussian integration would probably improve both of these characteristics with little loss of accuracy.

2. The present program treats a completely determined system of dimension 2n+1 by 2n+1, and then solves that system to find the collocation coefficients. This procedure has worked satisfactorily in this thesis, but may not work as well at higher frequencies where the final linear system of equations may be ill-conditioned. As an alternative, it is recommended that the boundary conditions be applied at more than n points, say m points, where m is twice or three times as many points, and that the least squares technique be used to determine the

the collocation coefficients which give the minimum square error over-all. This may be thought of as "sampling more data" in order to get more information about the unknown function. The present program could be easily modified in this regard by replacing the spanning function matrix, Q1ZINT, by a new matrix of the form

$$Q1ZINT' = X^T X$$

where X is the new m by n+1 (m > n+1) matrix, and replacing the present right-hand-side vector, Q1COF with

$$Q1COF' = X^T Y$$

where Y is the new m by 1 right-hand-side vector. An alternative would be to employ a prepackaged statistical linear regression routine after either modifying the routine to accept complex data, or transforming the present system into a larger system of real numbers only.

The new area in which the collocation method might be employed is the calculation of the potential flow about a staggered cascade. The method coud be employed to calculate both the potential in the channel and above the upper blade. The program presented has been designed to enable the

calculation of flow within the channel of a staggered cascade. Unfortunately, there was not enough time to extend the study to this case.

APPENDIX A

PROGRAM DESCRIPTION

This section describes the computer program used to calculate the interference solution to the Gorelov linearization for unsteady transonic flow in a channel. The program written in IBM Fortran IV with the basic structure outlined by Stevens [5]. The basic points are:

1. Organization of the program into small subroutines, each of which performs a specific task.
2. Transmission data to and from subroutines via a formal parameter argument list. No common statements are used.

The end objective is code which is both easy to modify and maintain.

Each subroutine is designed with optional diagnostic printing of its input and output. This is controlled by the parameter IPT. The diagnostic output is printed (only) if IPT > 0. Each routine accepts IPT, sets IOT = IPT - 1, and then passes IOT as the print parameter to routines it calls. By this method, diagnostic output can be "cascaded" to any desired level. Large initial values of IPT should be avoided because of the spectacular amount of output which can be generated by the double integrals within Q1DCOF.

1. Main Program; including subroutines READ and ABSA.

The basic structure is given in Figure A-1. MAIN calls READ to read input data and then ABSA to calculate the

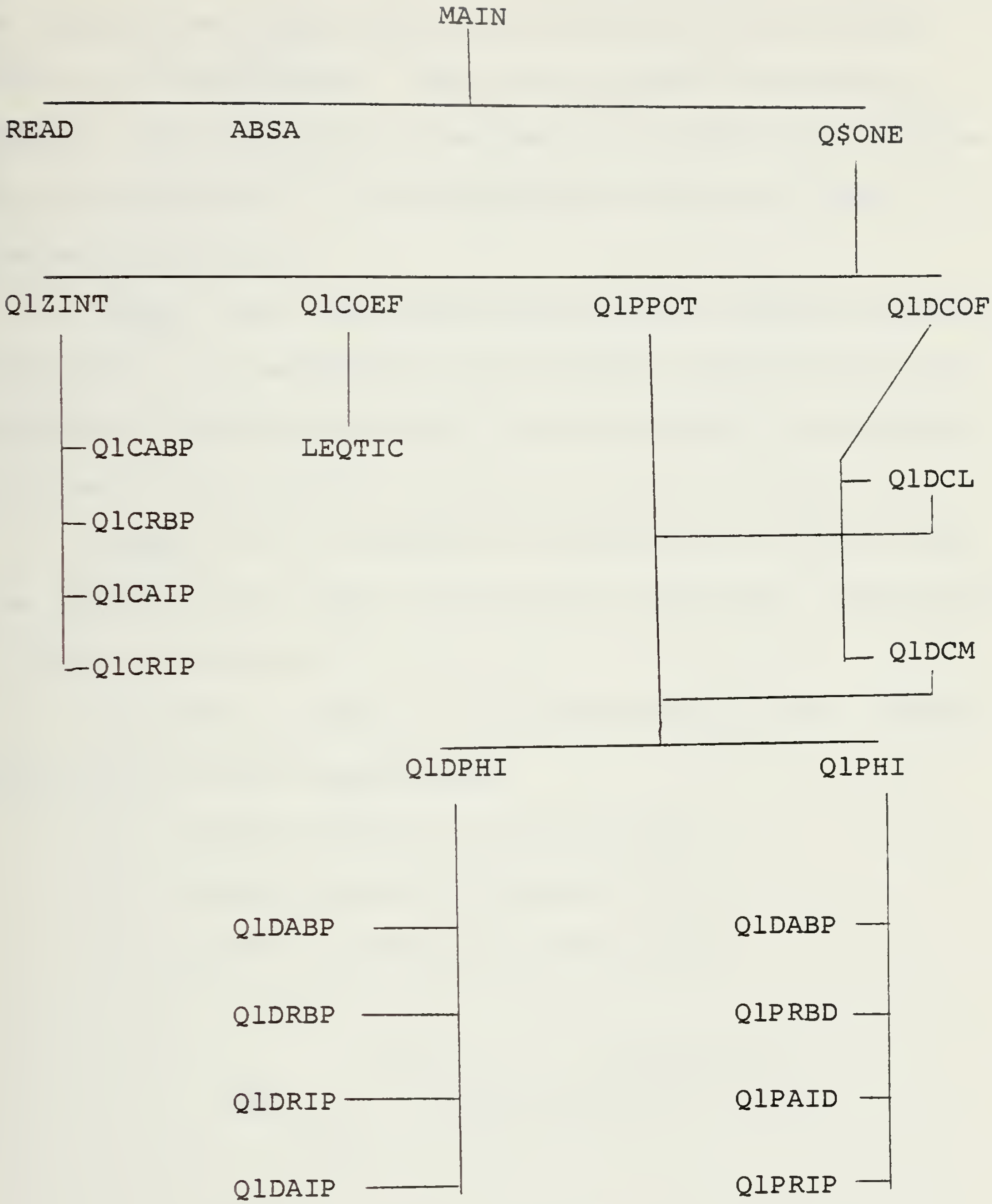


Figure A-1. Program Hierarchy

collocation points. The version of ABSA shown evenly spaces the collocation points across that portion of the blade subject to interference. ABSA may be easily replaced if different point spacing is desired, or if additional points are to be added for an overdetermined system and least squares approximation.

2. Q$ONE This subroutine controls the actual potential calculation. It performs no calculation itself, but calls subordinate subroutines where the calculations are actually performed. The calling hierarchy is shown in Figure A-1.

3. Q1ZINT This subroutine calculates the linear equation system arising from the boundary conditions. Hierarchy is shown in Figure A-2.

The matrix output is carried through Q1INT. Q1ZINT calls the following subprograms

a. Q1CRBP returns the value of ϕ_z^o

b. Q1CABP returns the value of $\phi_{z_1}^1$

c. Q1CRIP returns the values of ψ_z^o

$$\frac{\partial}{\partial z} \int_{-1+x_*}^{x-z} f_j(s) J_o[\omega\sqrt{(x-s)^2-z^2}]ds$$

where f_i is one of the set of i elementary functions, j=1,n

d. Q1CAIP returns the values of $\psi_{z_1}^1$

Q1ZINT

Q1CRBP: Calc. $\frac{\partial}{\partial z} \int v^{o}(s) J_{o}[\ ]$

Q1CABP: Calc. $\frac{\partial}{\partial z_1} \int v^{1} J_{o}[\ ]$

- Q1CWXM
  Sets end pts.
  - Q1CZXM
    Performs intgr.

Q1CRIP: Calc. $\frac{\partial}{\partial z} \int f_j J_o[\ ]_{z=0}$, $j=1,n$

Q1CAIP: Calc. $\frac{\partial}{\partial z_1} \int f_j J_o[\ ]_{z=0}$, $j=1,n$

- QICWSM
  Sets end pts.
  - Q1CZSM
    Performs intgr.
    - PLGNDR
      Spanning Fnctn.

Figure A-2

$$\frac{\partial}{\partial z_1} \int_{-1+OFFSET+x_*}^{x+z_1} f_j(s) J_o[\omega\sqrt{(x-s)^2-y_1^2}]ds$$

where f_j is one of the set of elementary functions. OFFSET is a parameter included to facilitate program conversion to a staggered cascade

e. PLGNDR is the subprogram which returns $f_j(x)$, the elementary spanning function. No other routine contains explicit reference to the spanning function. This facilitates easy replacement of the spanning functions should this be desired.

Q1CRBP and Q1CABP in turn call Q1CWXM and Q1CZXM. Q1CWXM computes end-points and then calls Q1CZXM, a complex integration routine based on SIMP by Shampine and Allen [6]. Q1CAIP and Q1CRIP call Q1CWSM and Q1CZSM to perform the integration. Q1ZINT passes the constant matrix to Q1COEF in the array Q1INT and the right-hand-side vector in the array Q1COF.

4. <u>Q1COEF</u> This subroutine employs the IMSL routine LEQT1C to solve the linear system received from Q1ZINT. The resulting coefficients are Q1ABCF for the adjacent blade and Q1RBCF for the reference blade. LEQT2C, the high precision complex IMSL routine may be directly substituted for LEQT1C. Q1COEF may be rewritten to employ the generalized inverse required for least squares approximation

$$A = (x^T x)^{-1} x^T y \quad \text{where} \quad x = \text{Q1INT}$$

$$y = \text{Q1COF}$$

after first performing the multiplication necessary to replace Q1INT and Q1COF with the proper matrix products in the call to LEQT1C.

5. Q1PPOT This subroutine calculates the potential, ϕ, and ϕ_x, at each collocation point along the reference blade but only if Q1PPOT receives a value of IPT > 0, requiring IPT $\geq$ 2 on input to the main program. If IPT $\leq$ 0, then the subroutine is exited before any calculations are performed. This subroutine is most useful for debugging Q1ZINT and Q1COEF. Q1PPOT calls Q1PABP, Q1PRBP, Q1PRIP, Q1PAIP, Q1DABP, Q1DRBP, Q1DRIP, and Q1DAIP, all of which will be described in the next section.

6. Q1DCOF This subroutine calculates the dimensionless coefficients of lift and moment; C_{ℓ_α}, C_{m_α}. Its internal hierarchy is shown in Figure A-3.

   a. Q1DCL calculates the nondimensional complex coefficient C_{ℓ_α}

   b. Q1DCM calculates the nondimensional complex coefficient C_{m_α}.

   c. Q1PRBP and Q1PABP calculate the potentials due to the reference and adjacent blades respectively. Q1PWXM and Q1PZXM are called to perform the actual integration.

d. Q1PRIP and Q1PAIP calculate the interference potentials along the reference and adjacent blades. Q1PZSM is called to perform the integration.

e. Q1DRBP, Q1DABP, Q1DRIP, and Q1DAIP correspond exactly to subroutines above except that the value returned is the partial derivative of the potential with respect to X. Q1DWXM, Q1DZSM, and Q1DXSM perform the co-reponding integrals.

6. Program Listing The program listing shown below incorporates the Legendre functions as spanning functions. Listings for a subroutine employing Gorelov's spanning function and the linear approximation program follow.

```
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Q,Z)
      DIMENSION X(13), Q2PT(13), Q2CP(13)
C     BLOCK ONE   READ AND EDIT DATA
      CALL ERRSET(208,256,-1,1)
C     WRITE (6,910)
    1 CALL READ(DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT)
C     IF(IPT.GT.0)WRITE(6,990)DK,DR,DW,SIGMA,OFFSET,RHO,N,NF
  910 FORMAT ('1',5X,'GORELOV CASCADE PROGRAM')
  990 FORMAT('0',5X,'INPUT VALUES TRANSMITTED TO MAIN PROGRAM:',/,
     1' ',10X,'DK',13X,'DR',13X,'DW    ',10X,'SIGMA',10X,'OFFSET',/,
     1' ',10X,5(E12.5,3X),/,
     2'0',10X,'RHO',12X,'N',5X,'NFCN'
     3,/,' ',10X,1(E12.5,3X), I2,4X,I2)
      CALL ABSA(N,OFFSET,X,RHO,DW    )
C
C
C     BLOCK TWO    CALLS CALCULATION ROUTINES FOR ZONE ONE AND ZONE TWO
      CALL Q$ONE(DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT,X)
      GO TO 1
      END
      SUBROUTINE READ (DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT)
      IMPLICIT REAL*8(A-H,O,P,R-Y)
      READ(5,905) DK,DR,DW,RHP,OFFSET,SIGMA,N,NF,IPT
      IF (DK.LT.0.0D0) STOP
      IF (IPT.GT.0) WRITE (6,910)
      GAMMA = 1.4D0
      DF = RHP * DSQRT((GAMMA + 1.0D0) * DW)
      RHO = DR
      IF(N.LE.13) GO TO  1
      N = 13
C     WRITE(6,935 )
    1 IF(OFFSET.LE.1.9D0) GO TO 2
      WRITE(6,940)
      OFFSET = OFFSET-1.0D0
      GO TO 1
    2 CONTINUE
C     IF (IPT.GT.0) WRITE (6,925) DK,DR,DW,SIGMA,OFFSET,RHO,N,NF,IPT
      RETURN
  905 FORMAT(6F10.4,3I2)
  910 FORMAT ('1',10X,'STAGGERED SUPERSONIC CASCADE PROGRAM')
  925 FORMAT ('0',10X,'DK',13X,'DR',13X,'DW    ',10X,'SIGMA',10X,'OFFSET'
     1,/,' ',10X,5(E12.5,3X),/,
     2'0',10X,'RHO',12X,'N',5X,'NFCN',2X,'IPT'
     3,/,' ',10X,1(E12.5,3X), I2,4X,I2,4X,I2)
  935 FORMAT('0',5X,'ORIGINAL N TOO LARGE (.GT.13) - RESET TO N = 13')
  940 FORMAT('0',5X,'OFFSET TOO LARGE (.GT.1.9), RESET AS OFFSET =
     1OFFSET - 1.0D0')
      END
```

```
      SUBROUTINE ABSA (N,OFFSET,X,DR,DW    )
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX*16(Q,Z)
      DIMENSION X(13)
      XINT = (2.0D0-DR)/DFLOAT(N+1)
      XL = DR - 1.0D0 + 1.0D-8
      DO 10 I = 1,N
      X(I) = XL + (XINT*DFLOAT(I))
      IF(X(I).EQ.0.0D0) X(I) = .1D-14
   10 CONTINUE
      RETURN
      END
      SUBROUTINE Q$ONE(DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT,X)
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Z,Q)
      DIMENSION Q1COF(26), Q1INT(26,26), Q1RBP(13), Q1ABP(13)
      DIMENSION Q1RBCF(13), Q1ABCF(13)
      DIMENSION X(13)
C     IF(IPT.GT.0)WRITE(6,990)DK,DR,RHO,OFFSET,SIGMA,N,NF,IPT
      ICT = IPT - 1
  990 FORMAT('0  SUBROUTINE Q$ONE ENTERED WITH:',/,
     1' ',5X,'DK',13X,'DR',13X,'RHO',12X,'OFFSET',9X,'SIGMA',10X,
     2'N',3X,'NF',2X,'IPT',/,
     3  5X,5(E12.5,3X),I2,2X,I2,2X,I3)
      CALL Q1ZINT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,ICT,Q1COF,Q1INT)
      CALL Q1COEF(Q1COF,Q1INT,N,IOT,Q1ABCF,Q1RBCF)
      CALL Q1PPCT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,Q1ABCF,Q1RBCF,ICT)
      I C T = 0
      CALL Q1CCCF(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,ICT)
      RETURN
      END
C
C
C
      SUBROUTINE Q1ZINT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,
     1      IPT,Q1COF,Q1INT)
      IMPLICIT REAL * 8(A-H,O,P,R-Y), COMPLEX * 16(Z,Q)
      DIMENSION Q1COF(26), Q1INT(26,26), QINTRP(13), QINTAP(13)
      DIMENSION  X(13)
  990 FORMAT ('0',10X,'Q1ZINT ENTERED WITH:',/,
     1 ' ',10X,'DK',13X,'DR',13X,'RHO',12X,'OFFSET',9X,'SIGMA',10X,
     2  'N',2X,'IPT',/,
     3  ' ',10X,5(E12.5,3X),I2,2X,I3)
      CCONST = CDEXP(DCMPLX(0.0D0,SIGMA))
      IF (IPT.GT.0) WRITE (6,990) DK,DR,RHO,OFFSET,SIGMA,N,IPT
      ICT = IPT - 1
      GAMMA = 1.4D0
      CLAMDA = DK/((GAMMA+1.0D0)*DW)
      DO 90 I = 1,N
      IN = I + N
```

```
      XSTN = X(I)
      XO = XSTN - DR
      CEXP = CDEXP(DCMPLX(0.0D0,DLAMDA*XSTN))
      CALL Q1CAIP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,QINTAP,N,IOT)
      CALL Q1CRIP(DK,DR,DW,RHO,OFFSET,XSTN,QINTRP,N,IOT)
      DO 20 J = 1,N
       JN = N + J
      J1 = J-1
      Q1INT(I,J)  = PLGNDR(XSTN,DR,J1) * CEXP
      Q1INT(IN,J) = QINTRP(J)
      Q1INT(I,JN) =   QINTAP(J)
      Q1INT(IN,JN) = PLGNDR(XSTN-OFFSET,DR,J1) * CEXP
   20 CONTINUE
      Q1COF(IN)  = -Q1CRBP(DK,DR,DW,RHO,XSTN,IOT)
      Q1COF(I)   = -Q1CABP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IOT)
   90 CONTINUE
      RETURN
      END
      COMPLEX FUNCTION Q1CRBP*16(DK,DR,DW,RHO,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION QINP(2)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1CRBP ENTERED WITH:',/,
     1   ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'XSTN',9X,'IPT'
     2   ,/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.RHO-1.0D0) GOTO 20
      IF(XSTN.GT.2.0D0)  GOTO 20
      ICT = IPT - 1
      QCK =  DCMPLX (0.0D0,DK)
      CALL Q1CWXM(DK,DR,DW,RHO,XSTN,QINP,IOT)
      Q1CRBP = -QCK*QINP(2) - QINP(1)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 Q1CRBP  = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0) RETURN
   30 WRITE (6,995) Q1CRBP
  995 FORMAT('0',10X,'Q1CRBP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1CABP*16(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION QINP(2)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1CABP ENTERED WITH:',/,
     1   ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,
     2   'SIGMA',8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFFSET
      IF(XASTN.LE.RHO-1.0D0) GOTO 20
```

```
      IF(XASTN.GT.2.0D0) GOTO 20                                        QLDR0725
      ICT = IPT - 1                                                     QLDR0730
      CDK = DCMPLX (0.0D0,DK)                                           QLDR0735
      QCONST = CDEXP( DCMPLX(0.0D0,SIGMA))                              QLDR0740
      CALL Q1CWXM(DK,DR,DW,RHO,XASTN,CINF,IOT)                          QLDR0745
      Q1CABP = -(CDK*QINP(2) + QINP(1)) * QCONST                        QLDR0750
      IF(IPT.LE.0) RETURN                                               QLDR0755
      GO TO 30                                                          QLDR0760
   20 Q1CABP = DCMPLX(0.0D0,0.0D0)                                      QLDR0765
      IF (IPT.LE.0) RETURN                                              QLDR0770
   30 WRITE (6,995) Q1CABP                                              QLDR0775
  995 FORMAT('0',10X,'Q1CABP = ',E14.7,',',E14.7)                       QLDR0780
      RETURN                                                            QLDR0785
      END                                                               QLDR0790
      SUBROUTINE Q1CWXM(DK,DR,DW,RHO,XSTN,QINP,IPT)                     QLDR0795
C                                                                       QLDR0800
C     N IS THE MAXIMUM DEGREE OF THE TERM U IN THE INTEGRALS            QLDR0805
C                                                                       QLDR0810
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Z,Q)               QLDR0815
      DIMENSION QINP(2)                                                 QLDR0820
      IF(IPT.GT.0) WRITE(6,990) DK,DR,RHO,XSTN,IPT                      QLDR0825
  990 FORMAT('0',10X,'Q1CWXM ENTERED WITH ARGUMENTS:',/,                QLDR0830
     1' ',10X,'DK',16X,'DR',16X,'RHO',15X,'XSTN',14X,'IPT',/,           QLDR0835
     2' ',10X,4(E13.6,5X),I2,5X,I3)                                     QLDR0840
      ICT = IPT - 1                                                     QLDR0845
      B = XSTN - RHO - 1.0D-10                                          QLDR0850
      A = -1.0D0                                                        QLDR0855
      CALL Q1CZXM(DK,DR,DW,RHO,XSTN,A,B,1,QANS,IOT)                     QLDR0860
      QINP(1) = QANS                                                    QLDR0865
      CALL Q1CZXM(DK,DR,DW,RHO,XSTN,A,B,2,QANS,IOT)                     QLDR0870
      QINP(2) = QANS                                                    QLDR0875
      IF(IPT.LE.0) RETURN                                               QLDR0880
      WRITE (6,995)                                                     QLDR0885
  995 FORMAT(' ',10X,'Q1CWXM RESULTS:',/,                               QLDR0890
     1' ',11X,'J',6X,'QINP(I)')                                         QLDR0895
      DO 1001 I = 1,2                                                   QLDR0900
  996 FORMAT(' ',10X,I3,5X,E12.6,',',E12.6)                             QLDR0905
 1001 WRITE (6,996) I, QINP(I)                                          QLDR0910
      RETURN                                                            QLDR0915
      END                                                               QLDR0920
      SUBROUTINE Q1CZXM(DK,DR,DW,RHO,XSTN,A,B,J,QANS,IPT)               QLDR0925
      IMPLICIT REAL*8(A - E,G,H,M,O,P,R - Y), COMPLEX*16(F,Q,Z)         QLDR0930
      DIMENSION FV(5),LORR(30),F1T(30),F2T(30),F3T(30),CAT(30),         QLDR0935
     1  ARESTT(30),QESTT(30),EPST(30),QPSUM(30)                         QLDR0940
      F(X) =  (X**J1)*CDEXP(QEXP*(X))                                   QLDR0945
     1       *(OMEGA*RHO/(DSQRT((XSTN-X)*(XSTN-X)-YY))*                 QLDR0950
     2       MMBSJ1(OMEGA*DSQRT((XSTN-X)*(XSTN-X)-YY),IER))             QLDR0955
      GAMMA = 1.4D0                                                     QLDR0960
```

```
      DM2 = (GAMMA + 1.0D0) * DW
      OMEGA = DSQRT(DK*DK*(1.0D0-DM2)/(DM2*DM2))
      YY = RHO * RHO
      CLAMDA = DK/DM2
      QEXP = DCMPLX(0.0D0,CLAMDA)
      J1 = J-1
      ACC = 1.0D-6
      U  =  9.0D-13
      IF (IPT.GT.0) WRITE(6,990)DK,DR,DW,RHO,XSTN,A,B,J,IPT
  990 FORMAT('0',15X,'Q1CZXM ENTERED WITH ARGUMENTS:',/,
     1' ',15X,'DK',13X,'DR',13X,'DW     ',10X,'RHO',12X,'XSTN',11X,
     2'A',14X,'B',14X,'J',2X,'IPT',/,
     3' ',15X,7(E14.7,','),I2,2X,I3)
      EFOURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      CERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      QPSUM(LVL) = 0.0
      ALPHA = A
      DA = B - A
      AREA = 0.0
      AREST = 0.0
      FV(1) = F(ALPHA)
      FV(3) = F(ALPHA + 0.5*DA)
      FV(5) = F(ALPHA + DA)
      KOUNT = 3
      WT = DA/6.0
      CEST = WT*(FV(1) + 4.0*FV(3) + FV(5))
    1 DX = 0.5*DA
      FV(2) = F(ALPHA + 0.5*DX)
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = CEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFOURU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.30)GO TO 5
      IF(KOUNT.GE.2000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
```

```
      F1T(LVL) = FV(3)                                                  QLDR1205
      F2T(LVL) = FV(4)                                                  QLDR1210
      F3T(LVL) = FV(5)                                                  QLDR1215
      DA = DX                                                           QLDR1220
      DAT(LVL) = DX                                                     QLDR1225
      AREST = ARESTL                                                    QLDR1230
      ARESTT(LVL) = ARESTR                                              QLDR1235
      QEST = QESTL                                                      QLDR1240
      QESTT(LVL) = QESTR                                                QLDR1245
      EPS = EPS/1.4                                                     QLDR1250
      EPST(LVL) = EPS                                                   QLDR1255
      FV(5) = FV(3)                                                     QLDR1260
      FV(3) = FV(2)                                                     QLDR1265
      GO TO 1                                                           QLDR1270
    2 QERROR = QERROR + QDIFF/15.0                                      QLDR1275
    3 IF(LORR(LVL).EQ.0)GO TO 4                                         QLDR1280
      QSUM = QPSUM(LVL) + QSUM                                          QLDR1285
      LVL = LVL - 1                                                     QLDR1290
      IF(LVL.GT.1)GO TO 3                                               QLDR1295
      QANS = QSUM - (B**J1) * CDEXP(QEXP*B)                             QLDR1300
      IF(IPT.GT.0) GO TO 11                                             QLDR1305
      IF(IER.EQ.129) GO TO 11                                           QLDR1310
      IF(IFLAG.EQ.1) RETURN                                             QLDR1315
      WRITE(6,990) DK,DR,DW,RHO,XSTN,A,B,J,IPT                          QLDR1320
   11 WRITE(6,995) QANS,IFLAG,IER,QERROR                                QLDR1325
  995 FORMAT(' ',15X,'RESULTS: QANS = ',E14.7,',',E14.7,/,              QLDR1330
     1' ',15X,'IFLAG',2X,'IER',5X,'QERROR',/,                           QLDR1335
     2' ',15X,I3,2X,I3,5X,E14.7,',',E14.7)                              QLDR1340
      RETURN                                                            QLDR1345
    4 QPSUM(LVL) = QSUM                                                 QLDR1350
      LORR(LVL) = 1                                                     QLDR1355
      ALPHA = ALPHA + DA                                                QLDR1360
      DA = DAT(LVL)                                                     QLDR1365
      FV(1) = F1T(LVL)                                                  QLDR1370
      FV(3) = F2T(LVL)                                                  QLDR1375
      FV(5) = F3T(LVL)                                                  QLDR1380
      AREST = ARESTT(LVL)                                               QLDR1385
      QEST = QESTT(LVL)                                                 QLDR1390
      EPS = EPST(LVL)                                                   QLDR1395
      GO TO 1                                                           QLDR1400
    5 IFLAG = 2                                                         QLDR1405
      GO TO 2                                                           QLDR1410
    6 IFLAG = 3                                                         QLDR1415
      GO TO 2                                                           QLDR1420
      END                                                               QLDR1425
C                                                                       QLDR1430
      SUBROUTINE Q1CAIP(DK,DR,DW,RHO,QFST,SGMA,XSTN,QCAIP,N,IPT)        QLDR1435
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)                 QLDR1440
```

```
      DIMENSION  CINP(13), QCAIP(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFST,SGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1CAIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFST',9X,'SGMA',9X
     2  ,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFST
      IF(XASTN.LE.DR+RHO-1.0D0) GO TO 20
      IF(XASTN.GT.2.0D0) GOTO 20
      ICT = IPT - 1
      QCCNST = CDEXP( DCMPLX(0.0D0,SGMA))
      CALL Q1CWSM(DK,DR,DW,RHO,XASTN,N,CINP,IOT)
      DO 10 I = 1,N
      QCAIP(I) = CINP(I)
   10 CONTINUE
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 ZERO = DCMPLX(0.0D0,0.0D0)
      DO 25 I = 1,N
      QCAIP(I) = ZERO
   25 CONTINUE
      IF(IPT.LE.0)RETURN
   30 WRITE (6,995)
  995 FORMAT ('0',10X,'Q1CAIP RESULTS: J      QCAIP(J)')
      DO 40 J = 1,N
      WRITE (6,996) J,QCAIP(J)
  996 FORMAT(' ',26X,I2,3X,E14.7,',',E14.7)
   40 CONTINUE
      RETURN
      END
      SUBROUTINE  Q1CRIP (DK,DR,DW,RHO,OFFSET,XSTN,QCRIP,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (C,Z)
      DIMENSION  QCRIP(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1CRIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,
     2  'XSTN',9X,'IPT'/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.DR+RHO+OFFSET-1.0D0) GOTO 20
      IF(XSTN.GT.2.0D0) GOTO 20
      ICT = IPT - 1
      CALL Q1CWSM(DK,DR,DW,RHO,XSTN,N,QCRIP,IOT)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 ZERO = DCMPLX(0.0D0,0.0D0)
      DO 25 I = 1,N
   25 QCRIP(I) = ZERO
      IF (IPT.LE.0) RETURN
   30 WRITE (6,995)
  995 FORMAT ('0',10X,'Q1CRIP RESULTS: J      QCRIP(J)')
```

```
      DO 40 J = 1,N
      WRITE (6,996) J,QCRIP(J)
  996 FORMAT(' ',26X,I2,3X,E14.7,',',E14.7)
   40 CONTINUE
      RETURN
      END
      SUBROUTINE Q1CWSM(DK,DR,DW,RHO, XSTN,N,QINF,IPT)
C
C     N IS THE MAXIMUM DEGREE OF THE TERM U IN THE INTEGRALS
C
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Z,Q)
      DIMENSION QINP(13)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,RHO,XSTN,N,IPT
  990 FORMAT('0',10X,'Q1CWSM ENTERED WITH ARGUMENTS:',/,
     1' ',10X,'DK',16X,'DR',16X,'RHO',15X,'XSTN',14X,'N',6X,'IPT',/,
     2' ',10X,4(E13.6,5X),I2,5X,I3)
      IOT = IPT - 1
      B = XSTN - RHO - 1.0D-10
      A = DR-1.0D0
      DO 30 J = 1,N
      CALL Q1CZSM(DK,DR,DW,RHO,XSTN,A,B,J,QANS,IOT)
      QINP(J) = QANS
   30 CONTINUE
      IF(IPT.LE.0) RETURN
      WRITE (6,995)
  995 FORMAT(' ',10X,'Q1CWSM RESULTS:',/,
     1' ',11X,'J',6X,'QINP(I)')
      DO 1001 I = 1,N
  996 FORMAT(' ',10X,I3,5X,E12.6,',',E12.6)
 1001 WRITE (6,996) I, QINP(I)
      RETURN
      END
      SUBROUTINE Q1CZSM(DK,DR,DW,RHO,XSTN,A,B,J,QANS,IPT)
      IMPLICIT REAL*8(A - E,G,H,M,O,P,R - Y), COMPLEX*16(F,Q,Z)
      DIMENSION FV(5),LORR(30),F1T(30),F2T(30),F3T(30),DAT(30),
     1  ARESTT(30),QESTT(30),EPST(30),QPSUM(30)
      F(X) = PLGNDR(X,DR,J1)*CDEXP(QEXP*(X))
     1      *(OMEGA*RHO/(DSQRT((XSTN-X)*(XSTN-X)-YY))*
     2      MMBSJ1(OMEGA*DSQRT((XSTN-X)*(XSTN-X)-YY),IER))
      GAMMA = 1.4D0
      DM2 = (GAMMA + 1.0D0) * DW
      OMEGA = DSQRT(DK*DK*(1.0D0-DM2)/(DM2*DM2))
      YY = RHO * RHO
      DLAMDA = DK/DM2
      QEXP = DCMPLX(0.0D0,DLAMDA)
      J1 = J-1
      ACC = 1.0D-6
      U   =  9.0D-13
```

```
      IF (IPT.GT.0) WRITE(6,990)DK,DR,CW,RHO,XSTN,A,B,J,IPT
  990 FORMAT('0',15X,'Q1CZSM ENTERED WITH ARGUMENTS:',/,
     1' ',15X,'DK',13X,'DR',13X,'CW    ',10X,'RHO',12X,'XSTN',11X,
     2'A',14X,'B',14X,'J',2X,'IPT',/,
     3' ',15X,7(E14.7,','),I2,2X,I3)
      EFOURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      CERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      CFSUM(LVL) = 0.0
      ALPHA = A
      DA = B - A
      AREA = 0.0
      AREST = 0.0
      FV(1) = F(ALPHA)
      FV(3) = F(ALPHA + 0.5*DA)
      FV(5) = F(ALPHA + DA)
      KOUNT = 3
      WT = DA/6.0
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))
    1 DX = 0.5*DA
      FV(2) = F(ALPHA + 0.5*DX)
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFOURU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.30)GO TO 5
      IF(KOUNT.GE.2000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
```

```
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      QANS = PLGNDR(B,DR,J1) * CDEXP(QEXP*B) - QSUM
      IF(IPT.GT.0) GO TO 11
      IF(IER.EQ.129) GO TO 11
      IF(IFLAG.EQ.1) RETURN
      WRITE(6,990) DK,DR,DW,RHC,XSTN,A,E,J,IPT
  11  WRITE(6,995) QANS,IFLAG,IER,QERROR
 995  FORMAT(' ',15X,'RESULTS: QANS = ',E14.7,',',E14.7,/,
    1' ',15X,'IFLAG',2X,'IER',5X,'QERROR',/,
    2' ',15X,I3,2X,I3,5X,E14.7,',',E14.7)
      RETURN
4     QPSUM(LVL) = QSUM
      LORR(LVL) = 1
      ALPHA = ALPHA + DA
      DA = DAT(LVL)
      FV(1) = F1T(LVL)
      FV(3) = F2T(LVL)
      FV(5) = F3T(LVL)
      AREST = ARESTT(LVL)
      QEST = QESTT(LVL)
      EPS = EPST(LVL)
      GO TO 1
5     IFLAG = 2
      GO TO 2
6     IFLAG = 3
      GO TO 2
      END
      REAL FUNCTION PLGNDR*8(X,DR,N)
      IMPLICIT REAL*8(A-H,O-Z)
      IF(N.EQ.0)GOTO 100
      X2 = X*X
      GOTO (101,102,103,104,105,106,107,108,109,110,111,112), N
 100  PLGNDR =  1.0D0
      RETURN
 101  PLGNDR =  X
      RETURN
 102  PLGNDR =  ((3.0D0)*X2-1.0D0)/2.0D0
      RETURN
```

```
103 PLGNDR =  (5.0D0*X2 - 3.0D0)*X/2.0D0                           QLDR2405
    RETURN                                                          QLDR2410
104 PLGNDR =  ((3.5D1*X2 - 3.0D1)*X2 + 3.0D0)/8.0D0                 QLDR2415
    RETURN                                                          QLDR2420
105 PLGNDR =  ((6.3D1*X2 - 7.0D1) *X2 +1.5D1)*X/8.0D0               QLDR2425
    RETURN                                                          QLDR2430
106 PLGNDR=(((231.0D0*X2-315.0D0)*X2+105.0D0)*X2-5.0D0)/16.0D0      QLDR2435
    RETURN                                                          QLDR2440
107 PLGNDR=(((429.0D0*X2-693.0D0)*X2+315.0D0)*X2-35.0D0)            QLDR2445
   1      *X/16.0D0                                                 QLDR2450
    RETURN                                                          QLDR2455
108 PLGNDR=((((6435.0D0*X2-1202.0D0)*X2+6930.0D0)*X2                QLDR2460
   1      -1260.0D0)*X2+35.0D0)/128.0D0                             QLDR2465
    RETURN                                                          QLDR2470
109 PLGNDR=((((12155.0D0*X2-25740.0D0)*X2+18018.0D0)                QLDR2475
   1      *X2-4620.0D0)*X2+315.0D0)/128.0D0                         QLDR2480
    RETURN                                                          QLDR2485
110 PLGNDR=(((((46189.0D0*X2-109395.0D0)*X2+90090.0D0)              QLDR2490
   1      *X2-30030.0D0)*X2-3465.0D0)*X2-63.0D0)/256.0D0            QLDR2495
    RETURN                                                          QLDR2500
111 PLGNDR=(((((88179.0D0*X2-230945.0D0)*X2+218790.0D0)*X2          QLDR2505
   1      -90090.0D0)*X2+15015.0D0)*X2-693.0D0)*X2/256.0D0          QLDR2510
    RETURN                                                          QLDR2515
112 PLGNDR=((((((676039.0D0*X2-1939938.0D0)*X2+2078505)*X2          QLDR2520
   1  -1021020.0D0)*X2+225225.0D0)*X2-18018.0D0)*X2-231.0D0)/1024.0D0 QLDR2525
    RETURN                                                          QLDR2530
    END                                                             QLDR2535
    SUBROUTINE Q1COEF( Q1COF,Q1INT,N,IPT,Q1ABCF,Q1RBCF)             QLDR2540
    IMPLICIT REAL * 8(A-H,O,P,R-Y), COMPLEX * 16 (Z,Q)              QLDR2545
    DIMENSION Q1COF(26), Q1INT(26,26), ZWA(300)                     QLDR2550
    DIMENSION Q1ABCF(13),Q1RBCF(13)                                 QLDR2555
    IF (IPT.GE.0) WRITE (6,90)                                      QLDR2560
    IE = 26                                                         QLDR2565
    M = 1                                                           QLDR2570
    N2 = 2*N                                                        QLDR2575
    IF (IPT.LE.0) GO TO 5                                           QLDR2580
    WRITE (6,98) N,N2                                               QLDR2585
 98 FORMAT('0',5X,'Q1COEF ENTERED WITH ',I2,' DEG  PWR SERIES (',I2, QLDR2590
   1 ' SQUARE MATRIX)')                                             QLDR2595
    DO 2 I = 1,N2                                                   QLDR2600
    WRITE(6,92) I, I, Q1COF(I)                                      QLDR2605
 92 FORMAT ('0', 10X, 'Q1COEF EQUATION SYSTEM, ROW ',I2,/,           QLDR2610
   1' ',10X,'Q1COF(',I2,') = ', E14.7,',',E14.7)                    QLDR2615
    DO 2 J = 1,N                                                    QLDR2620
    J2=J+N                                                          QLDR2625
    WRITE(6,91) I,J,Q1INT(I,J),I,J2,Q1INT(I,J2)                     QLDR2630
 91 FORMAT(' ',15X,2('QINT(',I2,',',I2,') = ',E14.7,',',E14.7,10X)) QLDR2635
  2 CONTINUE                                                        QLDR2640
```

```
5  IA = 26
   IJCB=0
   CALL LECT1C(Q1INT,N2,IA,Q1CCF,M,IE,IJOB,ZWA,IER)
   IF(IER.EQ.0) GOTO 30
   IF (IER.EQ.129) GC TO 10
   WRITE(6,93)
93 FORMAT( '0',10X,'Q1CCEF - ITERATIVE IMPROVEMENT FAILED, MATRIX TOO
  1 ILL-CONDITIONED. USE RESULTS WITH CAUTION.')
   GC TO 30
1C WRITE (6,95)
95 FORMAT('0',10X,'Q1CCEF - MATRIX ALGORITHMICALLY SINGULAR.  CCEFFIC
  1IENTS SET TC ZERO.')
   ZERO = DCMPLX(0.0DC,C.0D0)
   CC 20 I = 1,N
   I2 = I+N
   Q1COF(I) = ZERO
   Q1COF(I2) = ZERO
2C CCNTINUE
30 CCNTINUE
   CC 35 I = 1,N
   IN = I + N
   Q1ABCF(I) = Q1COF(IN)
   Q1RBCF(I) = Q1COF(I)
35 CCNTINUE
   IF(IPT.LE.0) RETURN
   IF (IPT.GE.0) WRITE (6,94)
   CC 40 I = 1,N
   IM1 = I-1
   WRITE (6,99) I, IM1, Q1RBCF(I), Q1ABCF(I)
40 CCNTINUE
99 FCRMAT('0',5X,I2,10X,I2,10X,2(E14.7,5X,E14.7,10X))
9C FCRMAT( '0',10X,'SUBROUTINE Q1CCEF -  COMPLEX POWER SERIES COEFFIC
  1IENTS')
94 FCRMAT( '0',5X,'INDEX',7X,'DEG PCLY',4X,'REFERENCE BLADE TERMS Q1C
  3CCF(INDEX)',7X,'ADJACENT BLADE TERMS Q1COF(2*INDEX)')
   RETURN
   END
   FUNCTION IQ1IFC(I)
C  .....................................................................
C
C  IQ1IFC IS A FUNCTION SUBROUTINE DESIGNED TC PRODUCE FACTORIALS
C  .....................................................................
   IF(I.EQ.0) GC TO 11
   IF(I.EQ.1) GC TO 11
   IQ1IFC=I
   II=I-1
   DC 10 J=1,II
10 IQ1IFC=IQ1IFC*(I-J)
```

```
      GO TO 12                                                          QLDR2885
   11 IC1IFC=1                                                          QLDR2890
   12 RETURN                                                            QLDR2895
      END                                                               QLDR2900
      SUBROUTINE Q1PPOT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,Q1ABCF,Q1RBCF,IPT)QLDR2905
      IMPLICIT REAL * 8(A-H,O,P,R-Y), COMPLEX * 16(Z,Q)                 QLDR2910
      DIMENSION Q1ABCF(13), Q1RBCF(13), QRR(13), QAA(13), QPHI(13)      QLDR2915
      DIMENSION QDRR(13),QDAA(13),QDPHI(13)                             QLDR2920
      DIMENSION X(13)                                                   QLDR2925
      IF(IPT.LE.0) RETURN                                               QLDR2930
      WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT                     QLDR2935
  990 FORMAT('1',10X,'Q1PPOT -CALCULATION OF COMPLEX POTENTIALS FOR ZONEQLDR2940
     1 ONE',/,'0',10X,'THESE CALCULATIONS ARE BASED ON THE SPANNING FUNCQLDR2945
     2TION COEFFICIENTS GIVEN DIRECTLY ABOVE',/                         QLDR2950
     3 ,' ',10X,'DK',13X,'DR',13X,'DK',13X,'RHO',12X,'OFFSET' ,9X,'SIGMA'QLDR2955
     4 ,10X,'N',3X,'IPT',/,' ',10X,                                     QLDR2960
     56(E12.5,3X),I2,2X,I3)                                             QLDR2965
   90 FORMAT ('0 X-STATION NUMBER ',I2,', XSTN = ',F6.4,/,              QLDR2970
     1'   BL TOTAL POTENTIAL',14X,'REF BL POTENTIAL',8X,                QLDR2975
     2 'ADJ BL POTENTIAL',8X,'REF BL INT POTENTIAL',5X,                 QLDR2980
     3 'ADJ BL INT POTENTIAL')                                          QLDR2985
   91 FORMAT('  R  ',2(E14.7,','),8(E11.4,','))                         QLDR2990
   92 FORMAT('  A  ',2(E14.7,','),8(E11.4,','))                         QLDR2995
      IOT = IPT - 1                                                     QLDR3000
      RZERO = 0.0D0                                                     QLDR3005
      DO 10 I = 1,N                                                     QLDR3010
      XSTN = X(I)                                                       QLDR3015
      QRO = Q1PRBP(DK,DR,DW,RZERO,XSTN,IOT)                             QLDR3020
      QRP = Q1PRBP(DK,DR,DW,RHO,XSTN,IOT)                               QLDR3025
      QAO = Q1PABP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,IOT)                QLDR3030
      QAP = Q1PABP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IOT)                  QLDR3035
      QRIPO = Q1PRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)           QLDR3040
      QRIPP = Q1PRIP(DK,DR,DW,RHO,OFFSET,XSTN,Q1RBCF,N,IOT)             QLDR3045
      QAIPO = Q1PAIP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)     QLDR3050
      QAIPP = Q1PAIP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)       QLDR3055
      QR = QRO + QAP + QRIPO + QAIPP                                    QLDR3060
      QA = QRP + QAO + QRIPP + QAIPO                                    QLDR3065
      QPHI(I)=Q1PHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,XSTN,IOT) QLDR3070
      WRITE (6,90) I,XSTN                                               QLDR3075
      WRITE(6,91) QR,QRO,QAP,QRIPO,QAIPP                                QLDR3080
      WRITE(6,92) QA,QRP,QAO,QRIPP,QAIPO                                QLDR3085
      QRR(I) = QR                                                       QLDR3090
      QAA(I) = QA                                                       QLDR3095
      QDRO = Q1DRBP(DK,DR,DW,RZERO,XSTN,IOT)                            QLDR3100
      QDRP = Q1DRBP(DK,DR,DW,RHO,XSTN,IOT)                              QLDR3105
      QDAO = Q1DABP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,IOT)               QLDR3110
      QDAP = Q1DABP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IOT)                 QLDR3115
      QDRIPO = Q1DRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)          QLDR3120
```

```
      QDRIPP = Q1DRIP(DK,DR,DW,RHO,OFFSET,XSTN,Q1RBCF,N,IOT)             QLDR3125
      QDAIPO = Q1DAIP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)     QLDR3130
      QDAIPP = Q1DAIP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)       QLDR3135
      QDR = QDRO + QDAF + QDRIPO + QDAIPP                                QLDR3140
      QDA = QDRP + QDAO + QDRIPP + QDAIPO                                QLDR3145
      QDPHI(I)=Q1DPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,         QLDR3150
     1      XSTN,IOT)                                                    QLDR3155
      WRITE (6,95) I,XSTN                                                QLDR3160
      WRITE(6,91) QDR,QDRO,QDAP,QDRIPO,QDAIPP                            QLDR3165
      WRITE(6,92) QDA,QDRP,QDAO,QDRIPP,QDAIPO                            QLDR3170
      QDRR(I) = QDR                                                      QLDR3175
      QDAA(I) = QDA                                                      QLDR3180
   95 FORMAT ('0 X-STATION NUMBER ',I2,', XSTN = ',F6.4,/,               QLDR3185
     1'  BL TOTAL D(POT)/DX',14X,'REF BL D(POT)/DX',8X,                  QLDR3190
     2 'ADJ BL D(POT)/DX',8X,'REF BL INT D(POT)/DX',5X,                  QLDR3195
     3 'ADJ BL INT D(POT)/DX')                                           QLDR3200
   10 CONTINUE                                                           QLDR3205
      WRITE(6,994)                                                       QLDR3210
  994 FORMAT ('1',10X,'SUMMARY LISTING',/,                               QLDR3215
     1'0',10X,'XSTN',7X,'SINGLE BLADE TOTAL POTENTIAL',6X,               QLDR3220
     1  'REF BLADE POTENTIAL',15X,'ADJ BLADE POTENTIAL')                 QLDR3225
      DO 20 I = 1,N                                                      QLDR3230
      XSTN = X(I)                                                        QLDR3235
      WRITE(6,94) XSTN,QPHI(I),QRR(I),QAA(I)                             QLDR3240
   94 FORMAT(' ',10X,F6.4,3(5X,E14.7,',',E14.7))                         QLDR3245
   20 CONTINUE                                                           QLDR3250
      WRITE(6,996)                                                       QLDR3255
  996 FORMAT('0',10X,'XSTN',7X,'SINGLE BLADE TOTAL D(POT)/DX',6X,        QLDR3260
     1  'REF BLADE D(POT)/DX',15X,'ADJ BLADE D(POT)/DX')                 QLDR3265
      DO 50 I = 1,N                                                      QLDR3270
      XSTN = X(I)                                                        QLDR3275
      WRITE(6,94) XSTN,QDPHI(I),QDRR(I),QDAA(I)                          QLDR3280
   50 CONTINUE                                                           QLDR3285
      RETURN                                                             QLDR3290
      END                                                                QLDR3295
      SUBROUTINE Q1DCOF(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IPT)   QLDR3300
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)                QLDR3305
      DIMENSION Q1ABCF(13), Q1RBCF(13)                                   QLDR3310
      IF(IPT.GT.0)WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT          QLDR3315
  990 FORMAT('1',10X,'Q1DCOF - CALCULATION OF COMPLEX DIMENSIONLESS AEROQLDR3320
     1DYNAMIC COEFFICIENTS',/,                                           QLDR3325
     3 '0',10X,'DK',13X,'DR',13X,'DW',13X,'RHO',12X,'OFFSET',9X,'SIGMA'  QLDR3330
     4 ,10X,'N',3X,'IPT',/,' ',10X,                                      QLDR3335
     56(E12.5,3X),I2,2X,I3)                                              QLDR3340
      ICT = IPT - 3                                                      QLDR3345
  999 QDCL = Q1DCL(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IOT)        QLDR3350
      QDCM = Q1DCM(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IOT)        QLDR3355
      GAMMA = 1.4D0                                                      QLDR3360
```

```
      TAU = (2.0D0*DSQRT((GAMMA + 1.0D0)*DW))/DR
      WRITE (6,90) DK,TAU,DW,N,SIGMA,CDCL,CDCM
 C    FORMAT('0',5X,'DK = ',F6.3,', TAU = ',F7.4,', DW = ',F6.3,', N = '
     1      ,I2,', SIGMA = ',F6.3,', CL = ',F9.4,',',F9.4,', CM = ',
     2      F9.4,',',F9.4)
      RETURN
      END
      COMPLEX FUNCTION Q1DCL*16(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBC
     1F,IPT)
      IMPLICIT REAL*8(A - E,G,H,O,P,R - Y), COMPLEX*16(F,Q,Z)
      DIMENSION Q1ABCF(13), Q1RBCF(13)
      DIMENSION FV(5),F1T(60),F2T(60),F3T(60),QESTT(60),QPSUM(60)
      DIMENSION DAT(60),  ARESTT(60),EPST(60)
      DIMENSION LORR(60)
      F(X) = (Q1DPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,ICT)
     1    -QALPHA*Q1PHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,IOT))
      IF (IPT.GT.0) WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
    0 FORMAT('0',10X,'Q1DCL ENTERED WITH:',/,
     3 ' ',10X,'DK',13X,'DR',13X,'DK',13X,'RHO',12X,'OFFSET',9X,'SIGMA'
     4 ,10X,'N',3X,'IPT',/,' ',10X,
     56(E12.5,3X),I2,2X,I3)
      A = - 1.0D0
      B = 1.0D0
      ICT = IPT - 1
      GAMMA = 1.4D0
      CLAMDA = DK/((GAMMA + 1.0D0) * DW)
      QALPHA = DCMPLX(0.0D0,CLAMDA-DK)
      U  =  9.0E - 13
      ACC = 1.0D-5
      EFOURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      QERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      QPSUM(LVL) = 0.0
      ALPHA = A
      DA = B - A
      AREA = 0.0
      AREST = 0.0
      FV(1) = F(ALPHA)
      FV(3) = F(ALPHA + 0.5*DA)
      FV(5) = F(ALPHA + DA)
      KOUNT = 3
      WT = DA/6.0
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))
      DX = 0.5*DA
      FV(2) = F(ALPHA + 0.5*DX)
```

```
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFCLRU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.60)GO TO 5
      IF(KOUNT.GE.4000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      Q1DCL = QSUM * 2.0D0
      IF(IPT.GT.0) GO TO 11
      IF(IFLAG.EQ.1) RETURN
      WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
11    WRITE(6,995) Q1DCL,IFLAG,QERROR
995   FORMAT(' ',15X,'RESULTS: Q1DCL = ',E14.7,',',E14.7,/,
     1' ',15X,'IFLAG',2X,'QERROR',/,
     2' ',15X,I3,2X,E14.7,',',E14.7)
      RETURN
4     QPSUM(LVL) = QSUM
      LORR(LVL) = 1
      ALPHA = ALPHA + DA
      DA = DAT(LVL)
```

```
      FV(1) = F1T(LVL)                                                  QLDR3845
      FV(3) = F2T(LVL)                                                  QLDR3850
      FV(5) = F3T(LVL)                                                  QLDR3855
      AREST = ARESTT(LVL)                                               QLDR3860
      QEST = QESTT(LVL)                                                 QLDR3865
      EPS = EPST(LVL)                                                   QLDR3870
      GO TO 1                                                           QLDR3875
    5 IFLAG = 2                                                         QLDR3880
      GO TO 2                                                           QLDR3885
    6 IFLAG = 3                                                         QLDR3890
      GO TO 2                                                           QLDR3895
      END                                                               QLDR3900
      COMPLEX FUNCTION Q1DCM*16(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCQLDR3905
     1F,IPT)                                                            QLDR3910
      IMPLICIT REAL*8(A - E,G,H,O,P,R - Y), COMPLEX*16(F,Q,Z)           QLDR3915
      DIMENSION Q1ABCF(13), Q1RBCF(13)                                  QLDR3920
      DIMENSION FV(5),F1T(60),F2T(60),F3T(60),QESTT(60),QFSUM(60)       QLDR3925
      DIMENSION DAT(60 ),  ARESTT(60),EPST(60)                          QLDR3930
      DIMENSION LORR(60)                                                QLDR3935
      F(X) = X*(Q1DPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,IOT) QLDR3940
     1   -QALPHA*Q1PHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,IOT))QLDR3945
      IF (IPT.GT.0) WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT       QLDR3950
  990 FORMAT('0',10X,'Q1DCM ENTERED WITH:',/,                           QLDR3955
     3 ' ',10X,'DK',13X,'DR',13X,'DW',13X,'RHO',12X,'OFFSET',9X,'SIGMA' QLDR3960
     4 ,10X,'N',3X,'IPT',/,' ',10X,                                     QLDR3965
     56(E12.5,3X),I2,2X,I3)                                             QLDR3970
      A = - 1.0D0                                                       QLDR3975
      B = 1.0D0                                                         QLDR3980
      ICT = IPT - 1                                                     QLDR3985
      GAMMA = 1.4D0                                                     QLDR3990
      CLAMDA = DK/((GAMMA + 1.0D0) * DW)                                QLDR3995
      QALPHA = DCMPLX(0.0D0,CLAMDA-DK)                                  QLDR4000
      U  =  9.0E - 13                                                   QLDR4005
      ACC = 1.0D-5                                                      QLDR4010
      EFOURU = 4.0*U                                                    QLDR4015
      IFLAG = 1                                                         QLDR4020
      EPS = ACC                                                         QLDR4025
      QERROR = DCMPLX(0.0D0,0.0D0)                                      QLDR4030
      LVL = 1                                                           QLDR4035
      LORR(LVL) = 1                                                     QLDR4040
      QFSUM(LVL) = 0.0                                                  QLDR4045
      ALPHA = A                                                         QLDR4050
      DA = B - A                                                        QLDR4055
      AREA = 0.0                                                        QLDR4060
      AREST = 0.0                                                       QLDR4065
      FV(1) = F(ALPHA)                                                  QLDR4070
      FV(3) = F(ALPHA + 0.5*DA)                                         QLDR4075
      FV(5) = F(ALPHA + DA)                                             QLDR4080
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```
      KOUNT = 3                                                         QLDR4085
      WT = DA/6.0                                                       QLDR4090
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))                             QLDR4095
    1 DX = 0.5*DA                                                       QLDR4100
      FV(2) = F(ALPHA + 0.5*DX)                                         QLDR4105
      FV(4) = F(ALPHA + 1.5*DX)                                         QLDR4110
      KOUNT = KOUNT + 2                                                 QLDR4115
      WT = DX/6.0                                                       QLDR4120
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))                            QLDR4125
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))                            QLDR4130
      QSUM = QESTL + QESTR                                              QLDR4135
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))      QLDR4140
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))      QLDR4145
      AREA = AREA + ((ARESTL + ARESTR) - AREST)                         QLDR4150
      QDIFF = QEST - QSUM                                               QLDR4155
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2                         QLDR4160
      IF(DABS(DX).LE.EFCURU*DABS(ALPHA))GO TO 5                         QLDR4165
      IF(LVL.GE.60)GO TO 5                                              QLDR4170
      IF(KOUNT.GE.4000)GO TO 6                                          QLDR4175
      LVL = LVL + 1                                                     QLDR4180
      LORR(LVL) = 0                                                     QLDR4185
      F1T(LVL) = FV(3)                                                  QLDR4190
      F2T(LVL) = FV(4)                                                  QLDR4195
      F3T(LVL) = FV(5)                                                  QLDR4200
      DA = DX                                                           QLDR4205
      DAT(LVL) = DX                                                     QLDR4210
      AREST = ARESTL                                                    QLDR4215
      ARESTT(LVL) = ARESTR                                              QLDR4220
      QEST = QESTL                                                      QLDR4225
      QESTT(LVL) = QESTR                                                QLDR4230
      EPS = EPS/1.4                                                     QLDR4235
      EPST(LVL) = EPS                                                   QLDR4240
      FV(5) = FV(3)                                                     QLDR4245
      FV(3) = FV(2)                                                     QLDR4250
      GO TO 1                                                           QLDR4255
    2 QERROR = QERROR + QDIFF/15.0                                      QLDR4260
    3 IF(LORR(LVL).EQ.0)GO TO 4                                         QLDR4265
      QSUM = QPSUM(LVL) + QSUM                                          QLDR4270
      LVL = LVL - 1                                                     QLDR4275
      IF(LVL.GT.1)GO TO 3                                               QLDR4280
      Q1DCM = QSUM * 2.0D0                                              QLDR4285
      IF(IPT.GT.0) GO TO 11                                             QLDR4290
      IF(IFLAG.EQ.1) RETURN                                             QLDR4295
      WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT                      QLDR4300
   11 WRITE(6,995) Q1DCM,IFLAG,QERROR                                   QLDR4305
  995 FORMAT(' ',15X,'RESULTS: Q1DCM = ',E14.7,',',E14.7,/,             QLDR4310
     1' ',15X,'IFLAG',2X,'QERROR',/,                                    QLDR4315
     2' ',15X,I3,2X,E14.7,',',E14.7)                                    QLDR4320
```

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      RETURN                                                            QLDR4325
4     QPSUM(LVL) = QSUM                                                 QLDR4330
      LORR(LVL) = 1                                                     QLDR4335
      ALPHA = ALPHA + DA                                                QLDR4340
      DA = DAT(LVL)                                                     QLDR4345
      FV(1) = F1T(LVL)                                                  QLDR4350
      FV(3) = F2T(LVL)                                                  QLDR4355
      FV(5) = F3T(LVL)                                                  QLDR4360
      AREST = ARESTT(LVL)                                               QLDR4365
      QEST = QESTT(LVL)                                                 QLDR4370
      EPS = EPST(LVL)                                                   QLDR4375
      GO TO 1                                                           QLDR4380
5     IFLAG = 2                                                         QLDR4385
      GO TO 2                                                           QLDR4390
6     IFLAG = 3                                                         QLDR4395
      GO TO 2                                                           QLDR4400
      END                                                               QLDR4405
      COMPLEX FUNCTION Q1PHI*16 (DK,DR,DW,RHO,OFFSET,SIGMA,             QLDR4410
     1     N,Q1ABCF,Q1RBCF,XSTN,IPT)                                    QLDR4415
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Q,Z)                     QLDR4420
      DIMENSION Q1ABCF(13), Q1RBCF(13)                                  QLDR4425
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT     QLDR4430
990   FORMAT('0',10X,'Q1PHI ENTERED WITH:',/,                           QLDR4435
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,'SIGMA'QLDR4440
     2  ,8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)                   QLDR4445
      RZERO = 0.0D0                                                     QLDR4450
      ICT = IPT - 1                                                     QLDR4455
      Q1PHI = Q1PRBP(DK,DR,DW,RZERO,XSTN,IOT)                           QLDR4460
     1 + Q1PRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)                QLDR4465
     2 + Q1PABP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,IOT)                      QLDR4470
     3 + Q1PAIP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,Q1ABCF,N,ICT)             QLDR4475
      IF(IPT.LE.0) RETURN                                               QLDR4480
      WRITE (6,995) Q1PHI                                               QLDR4485
995   FORMAT('0',10X,'Q1PHI = ',E14.7,',',E14.7)                        QLDR4490
      RETURN                                                            QLDR4495
      END                                                               QLDR4500
      COMPLEX FUNCTION Q1PRBP*16(DK,DR,DW,RHO,XSTN,IPT)                 QLDR4505
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)               QLDR4510
      DIMENSION QINP(2)                                                 QLDR4515
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT                  QLDR4520
990   FORMAT('0',10X,'Q1PRBP ENTERED WITH:',/,                          QLDR4525
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'XSTN',9X,'IPT'    QLDR4530
     2  ,/,' ',10X,5(E12.5,','),I3)                                     QLDR4535
      IF(XSTN.LE.RHO-1.0D0) GOTO 20                                     QLDR4540
      ICT = IPT - 1                                                     QLDR4545
      QDK = DCMPLX (0.0D0,DK)                                           QLDR4550
      CALL Q1PWXM(DK,DR,DW,RHO,XSTN,QINF,IOT)                           QLDR4555
      Q1PRBP = -QDK*QINF(2) - QINP(1)                                   QLDR4560
```

```
      IF(IPT.LE.0) RETURN
      GOTO 60
   20 Q1PRBP = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0) RETURN
   60 WRITE (6,995) Q1PRBP
  995 FORMAT('0',10X,'Q1PRBP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1PABP*16(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION QINP(2)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1PABP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,
     2  'SIGMA',8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFFSET
      IF(XASTN.LE.RHO-1.0D0) GOTO 20
      IOT = IPT - 1
      QDK = DCMPLX (0.0D0,DK)
      QCONST = CDEXP( DCMPLX(0.0D0,SIGMA))
      CALL Q1PWXM(DK,DR,DW,RHO,XASTN,QINP,IOT)
      Q1PABP = (QDK*QINP(2) + QINP(1)) * QCONST
      IF(IPT.LE.0) RETURN
      GO TO 60
   20 Q1PABP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   60 WRITE (6,995) Q1PABP
  995 FORMAT ('0', 10X,'Q1PABP = ',E14.7,',',E14.7)
      RETURN
      END
      SUBROUTINE Q1PWXM(DK,DR,DW,RHO,XSTN,QINP,IPT)
C
C     N IS THE MAXIMUM DEGREE OF THE TERM U IN THE INTEGRALS
C
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Z,Q)
      DIMENSION QINP(2)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1PWXM ENTERED WITH ARGUMENTS:',/,
     1' ',10X,'DK',16X,'DR',16X,'RHO',15X,'XSTN',14X,'IPT',/,
     2' ',10X,4(E13.6,5X),I2,5X,I3)
      IOT = IPT - 1
      B = XSTN - RHO - 1.0D-10
      A = -1.0D0
      CALL Q1PZXM(DK,DR,DW,RHO,XSTN,A,B,1,QANS,IOT)
      QINP(1) = QANS
      CALL Q1PZXM(DK,DR,DW,RHO,XSTN,A,B,2,QANS,IOT)
      QINP(2) = QANS
      IF(IPT.LE.0) RETURN
```

```
      WRITE (6,995)                                                     QLDR4805
  995 FORMAT(' ',10X,'Q1PWXM RESULTS:',/,                               QLDR4810
     1' ',11X,'J',6X,'QINP(I)')                                         QLDR4815
      DO 1001 I = 1,2                                                   QLDR4820
  996 FORMAT(' ',10X,I3,5X,E12.6,',',E12.6)                             QLDR4825
 1001 WRITE (6,996) I, QINP(I)                                          QLDR4830
      RETURN                                                            QLDR4835
      END                                                               QLDR4840
      SUBROUTINE Q1PZXM (DK,DR,DW,RHO,XSTN,A,B,J,QANS,IPT)              QLDR4845
      IMPLICIT REAL*8 (A - E,G,H,M,O,P,R - Y), COMPLEX*16(F,Q,Z)        QLDR4850
      DIMENSION FV(5),LCRR(60),F1T(60),F2T(60),F3T(60),DAT(60),         QLDR4855
     1  ARESTT(60),QESTT(60),EPST(60),QFSUM(60)                         QLDR4860
      F(X) = (X**J1)*CDEXP(QEXP*(X))                                    QLDR4865
     1      * MMESJO((OMEGA * DSQRT( (XSTN-X)*(XSTN-X) - YY)), IER)     QLDR4870
      GAMMA = 1.4D0                                                     QLDR4875
      YY = RHO * RHO                                                    QLDR4880
      DM2 = (GAMMA + 1.0D0) * DW                                        QLDR4885
      OMEGA = DSQRT (DK*DK*(1.0D0-DM2)/(DM2*DM2))                       QLDR4890
      DLAMDA = DK/DM2                                                   QLDR4895
      DM = DSQRT(DM2)                                                   QLDR4900
      Q2EXP = CDEXP (DCMPLX (0.0D0, -DLAMDA*XSTN))                      QLDR4905
      QEXP = DCMPLX(0.0D0,DLAMDA)                                       QLDR4910
      J1 = J - 1                                                        QLDR4915
      U  =  9.0D-13                                                     QLDR4920
      ACC = 1.0D-6                                                      QLDR4925
      IF (IPT.GT.0) WRITE(6,990)DK,DR,DW,RHO,XSTN,A,B,J,IPT             QLDR4930
  990 FORMAT('0',15X,'Q1PZXM ENTERED WITH ARGUMENTS:',/,                QLDR4935
     1' ',15X,'DK',13X,'DR',13X,'DW    ',10X,'RHO',12X,'XSTN',11X,      QLDR4940
     2'A',14X,'B',14X,'J',2X,'IPT',/,                                   QLDR4945
     3' ',15X,7(E14.7,','),I2,2X,I3)                                    QLDR4950
      EPCURU = 4.0*U                                                    QLDR4955
      IFLAG = 1                                                         QLDR4960
      EPS = ACC                                                         QLDR4965
      QERROR = DCMPLX(0.0D0,0.0D0)                                      QLDR4970
      LVL = 1                                                           QLDR4975
      LCRR(LVL) = 1                                                     QLDR4980
      QFSUM(LVL) = 0.0                                                  QLDR4985
      ALPHA = A                                                         QLDR4990
      DA = B - A                                                        QLDR4995
      AREA = 0.0                                                        QLDR5000
      AREST = 0.0                                                       QLDR5005
      FV(1) = F(ALPHA)                                                  QLDR5010
      FV(3) = F(ALPHA + 0.5*DA)                                         QLDR5015
      FV(5) = F(ALPHA + DA)                                             QLDR5020
      KOUNT = 3                                                         QLDR5025
      WT = DA/6.0                                                       QLDR5030
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))                             QLDR5035
    1 DX = 0.5*DA                                                       QLDR5040
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```
      FV(2) = F(ALPHA + 0.5*DX)
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFOURU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.60)GO TO 5
      IF(KOUNT.GE.2000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      QANS = QSUM * Q2EXP / DM
      IF(IPT.GT.0) GO TO 11
      IF(IFLAG.EQ.1) RETURN
      WRITE(6,990) DK,DR,DW,RHO,XSIN,A,E,J,IPT
11    WRITE(6,995) QANS,IFLAG,IER,QERROR
995   FORMAT(' ',15X,'RESULTS: QANS = ',E14.7,',',E14.7,/,
     1' ',15X,'IFLAG',2X,'IER',5X,'QERROR',/,
     2' ',15X,I3,2X,I3,5X,E14.7,',',E14.7)
      RETURN
4     QPSUM(LVL) = QSUM
      LORR(LVL) = 1
      ALPHA = ALPHA + DA
```

```
      DA = CAT(LVL)
      FV(1) = F1T(LVL)
      FV(3) = F2T(LVL)
      FV(5) = F3T(LVL)
      AREST = ARESTT(LVL)
      QEST = QESTT(LVL)
      EPS = EPST(LVL)
      GO TO 1
5     IFLAG = 2
      GO TO 2
6     IFLAG = 3
      GO TO 2
      END
      COMPLEX FUNCTION Q1PAIP*16(DK,DR,DW,RHO,OFST,SGMA,XSTN,Q1CF,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION Q1CF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFST,SGMA,XSTN,IPT
990   FORMAT('0',10X,'Q1PAIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFST',9X,'SGMA',9X
     2  ,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFST
      IF(XASTN.LE.DR+RHO-1.0D0) GO TO 20
      A = DR-1.0D0
      B = XASTN - RHO - 1.0D-8
      ICT = IPT - 1
      QCONST = CDEXP( DCMPLX(0.0D0,SGMA))
      CALL Q1PZSM(DK,DR,DW,RHO,XASTN,A,B,N,QANS,Q1CF,ICT)
      Q1PAIP = QANS
      IF(IPT.LE.0) RETURN
      GOTO 60
20    Q1PAIP = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0)RETURN
60    WRITE (6,995) Q1PAIP
995   FORMAT('0',10X,'Q1PAIP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1PRIP*16(DK,DR,DW,RHO,OFFSET,XSTN,Q1CF,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION Q1CF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
990   FORMAT('0',10X,'Q1PRIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,
     2  'XSTN',9X,'IPT'/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.DR+RHO+OFFSET-1.0D0) GOTO 20
      A = DR-1.0D0
      B = XSTN - RHO - 1.0D-8
      ICT = IPT - 1
      CALL Q1PZSM(DK,DR,DW,RHO,XSTN,A,B,N,QANS,Q1CF,ICT)
```

```
      Q1PRIP = -QANS
      IF(IPT.LE.0) RETURN
      GOTO 60
   20 Q1PRIP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   60 WRITE (6,995) Q1PRIP
  995 FORMAT('0',10X,'Q1PRIP = ',E14.7,',',E14.7)
      RETURN
      END
      SUBROUTINE Q1PZSM (DK,DR,DW,RHO,XSTN,A,B,J,QANS,Q1CF,IPT)
      IMPLICIT REAL*8 (A - E,G,H,M,O,P,R - Y), COMPLEX*16(F,Q,Z)
      DIMENSION Q1CF(13)
      DIMENSION FV(5),LORR(60),F1T(60),F2T(60),F3T(60),DAT(60),
     1  ARESTT(60),QESTT(60),EPST(60),QPSUM(60)
      F(X) = CLGNDR(X,DR,J,Q1CF)*CDEXP(QEXP*(X))
     1      * MMBSJC((OMEGA * DSQRT( (XSTN-X)*(XSTN-X)  - YY)), IER)
      GAMMA = 1.4D0
      YY = RHO * RHO
      DM2 = (GAMMA + 1.0D0) * DW
      OMEGA = DSQRT (DK*DK*(1.0D0-DM2)/(DM2*DM2))
      DLAMDA = DK/DM2
      DM = DSQRT(DM2)
      Q2EXP = CDEXP (DCMPLX (0.0D0, -DLAMDA*XSTN))
      QEXP = DCMPLX(0.0D0,DLAMDA)
      U  =  9.0D-13
      ACC = 1.0D-6
      IF (IPT.GT.0) WRITE(6,990)DK,DR,DW,RHO,XSTN,A,B,J,IPT
  990 FORMAT('0',15X,'Q1PZSM ENTERED WITH ARGUMENTS:',/,
     1' ',15X,'DK',13X,'DR',13X,'DW   ',10X,'RHO',12X,'XSTN',11X,
     2'A',14X,'B',14X,'J',2X,'IPT',/,
     3' ',15X,7(E14.7,','),I2,2X,I3)
      EFOURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      QERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      QPSUM(LVL) = 0.0
      ALPHA = A
      DA = B - A
      AREA = 0.0
      AREST = 0.0
      FV(1) = F(ALPHA)
      FV(3) = F(ALPHA + 0.5*DA)
      FV(5) = F(ALPHA + DA)
      KOUNT = 3
      WT = DA/6.0
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))
```

```
1     DX = 0.5*DA
      FV(2) = F(ALPHA + 0.5*DX)
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EPDURU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.60)GO TO 5
      IF(KOUNT.GE.2000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      QANS = QSUM * Q2EXP / DM
      IF(IPT.GT.0) GO TO 11
      IF(IFLAG.EQ.1) RETURN
      WRITE(6,990) DK,DR,DW,RHO,XSTN,A,E,J,IPT
11    WRITE(6,995) QANS,IFLAG,IER,QERROR
995   FORMAT(' ',15X,'RESULTS: QANS = ',E14.7,' ',E14.7,/,
     1' ',15X,'IFLAG',2X,'IER',5X,'QERROR',/,
     2' ',15X,I3,2X,I3,5X,E14.7,' ',E14.7)
      RETURN
4     QPSUM(LVL) = QSUM
      LORR(LVL) = 1
```

```
      ALPHA = ALPHA + DA                                                QLDR6005
      DA = DAT(LVL)                                                     QLDR6010
      FV(1) = F1T(LVL)                                                  QLDR6015
      FV(3) = F2T(LVL)                                                  QLDR6020
      FV(5) = F3T(LVL)                                                  QLDR6025
      AREST = ARESTT(LVL)                                               QLDR6030
      QEST = QESTT(LVL)                                                 QLDR6035
      EPS = EPST(LVL)                                                   QLDR6040
      GO TO 1                                                           QLDR6045
    5 IFLAG = 2                                                         QLDR6050
      GO TO 2                                                           QLDR6055
    6 IFLAG = 3                                                         QLDR6060
      GO TO 2                                                           QLDR6065
      END                                                               QLDR6070
      COMPLEX FUNCTION QLGNDR*16(X,DR,N,Q1CF)                           QLDR6075
      IMPLICIT REAL * 8 (A-H,O,P,R-Y),  COMPLEX * 16(Q,Z)               QLDR6080
      DIMENSION Q1CF(13)                                                QLDR6085
      ZSUM = 0.0D0                                                      QLDR6090
      DO 10 J1 = 1,N                                                    QLDR6095
      J= J1 - 1                                                         QLDR6100
      ZSUM = ZSUM + PLGNDR(X,DR,J) * Q1CF(J1)                           QLDR6105
   10 CONTINUE                                                          QLDR6110
      QLGNDR = ZSUM                                                     QLDR6115
      RETURN                                                            QLDR6120
      END                                                               QLDR6125
      COMPLEX FUNCTION Q1DPHI*16 (DK,DR,DW,RHO,OFFSET,SIGMA,            QLDR6130
     1    N,Q1ABCF,Q1RBCF,XSTN,IPT)                                     QLDR6135
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Q,Z)                     QLDR6140
      DIMENSION Q1ABCF(13),Q1RBCF(13)                                   QLDR6145
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT     QLDR6150
  990 FORMAT('0',10X,'Q1DPHI ENTERED WITH:',/,                          QLDR6155
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,'SIGMA'QLDR6160
     2  ,8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)                   QLDR6165
      RZERO = 0.0D0                                                     QLDR6170
      IOT = IPT - 1                                                     QLDR6175
      Q1DPHI = Q1DREP(DK,DR,DW,RZERO,XSTN,IOT)                          QLDR6180
     1 + Q1DRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)                QLDR6185
     2 + Q1DAEP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,IOT)                      QLDR6190
     3 + Q1DAIP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)             QLDR6195
      IF(IPT.LE.0) RETURN                                               QLDR6200
      WRITE (6,995) Q1DPHI                                              QLDR6205
  995 FORMAT('0',10X,'Q1DPHI = ',E14.7,',',E14.7)                       QLDR6210
      RETURN                                                            QLDR6215
      END                                                               QLDR6220
      COMPLEX FUNCTION Q1DREP*16(DK,DR,DW,RHO,XSTN,IPT)                 QLDR6225
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)               QLDR6230
      DIMENSION QINP(2)                                                 QLDR6235
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT                  QLDR6240
```

```
 990 FORMAT('0',10X,'Q1DRBP ENTERED WITH:',/,
    1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'XSTN',9X,'IPT'
    2  ,/,' ',10X,5(E12.5,','),I3)
     IF(XSTN.LE.RHO-1.0D0) GOTO 20
     ICT = IPT - 1
     QDK = DCMPLX (0.0D0,DK)
     CALL Q1DWXM(DK,DR,DW,RHO,XSTN,QINP,IOT)
     Q1DRBP = -QDK*QINP(2) - QINP(1)
     IF(IPT.LE.0) RETURN
     GOTO 60
  20 Q1DRBP = DCMPLX(0.0D0,0.0D0)
     IF(IPT.LE.0) RETURN
  60 WRITE (6,995) Q1DRBP
 995 FORMAT('0',10X,'Q1DRBP = ',E14.7,',',E14.7)
     RETURN
     END
     COMPLEX FUNCTION Q1DABP*16(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT)
     IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
     DIMENSION QINP(2)
     IF(IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
 990 FORMAT('0',10X,'Q1DABP ENTERED WITH:',/,
    1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,
    2  'SIGMA',8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
     XASTN = XSTN - OFFSET
     IF(XASTN.LE.RHO-1.0D0) GO TO 20
     ICT = IPT - 1
     QDK = DCMPLX (0.0D0,DK)
     QCONST = CDEXP( DCMPLX(0.0D0,SIGMA))
     CALL Q1DWXM(DK,DR,DW,RHO,XASTN,QINP,IOT)
     Q1DABP = (QDK*QINP(2) + QINP(1)) * QCONST
     IF(IPT.LE.0) RETURN
     GO TO 60
  20 Q1DABP = DCMPLX(0.0D0,0.0D0)
     IF (IPT.LE.0) RETURN
  60 WRITE (6,995) Q1DABP
 995 FORMAT('0',10X,'Q1DABP = ',E14.7,',',E14.7)
     RETURN
     END
     SUBROUTINE Q1DWXM(DK,DR,DW,RHO, XSTN,QINP,IPT)
C
C    N IS THE MAXIMUM DEGREE OF THE TERM U IN THE INTEGRALS
C
     IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Z,Q)
     DIMENSION QINP(2)
     IF(IPT.GT.0) WRITE(6,990) DK,DR,RHO,XSTN,IPT
 990 FORMAT('0',10X,'Q1DWXM ENTERED WITH ARGUMENTS:',/,
    1' ',10X,'DK',16X,'DR',16X,'RHO',15X,'XSTN',14X,'IPT',/,
    2' ',10X,4(E13.6,5X),I2,5X,I3)
```

```
      ICT = IPT - 1
      B = XSTN - RHO - 1.0D-10
      A = -1.0D0
      CALL Q1DZXM(DK,DR,DW,RHO,XSTN,A,B,1,QANS,IOT)
      QINP(1) = QANS
      CALL Q1DZXM(DK,DR,DW,RHO,XSTN,A,B,2,QANS,IOT)
      QINP(2) = QANS
      IF(IPT.LE.0) RETURN
      WRITE (6,995)
  995 FORMAT(' ',10X,'Q1DWXM RESULTS:',/,
     1' ',11X,'J',6X,'QINP(I)')
      DO 1001 I = 1,2
  996 FORMAT(' ',10X,I3,5X,E12.6,',',E12.6)
 1001 WRITE (6,996) I, QINP(I)
      RETURN
      END
      SUBROUTINE Q1DZXM (DK,DR,DW,RHO,XSTN,A,B,J,QANS,IFT)
      IMPLICIT REAL*8(A - E,G,H,M,O,P,R - Y), COMPLEX*16(F,Q,Z)
      DIMENSION FV(5),LORR(60),F1T(60),F2T(60),F3T(60),DAT(60),
     1  ARESTT(60),QESTT(60),EPST(60),QPSUM(60)
      F(X) = (X**J1)*CDEXP(QEXP*(X))
     1      * MMBESJ1((OMEGA * DSQRT( (XSTN-X)*(XSTN-X) - YY)), IER)
     2      * OMEGA * (XSTN-X)/(DSQRT((XSTN-X)*(XSTN-X)-YY))
      GAMMA = 1.4D0
      YY = RHO * RHO
      DM2 = (GAMMA + 1.0D0) * DW
      OMEGA = DSQRT (DK*DK*(1.0D0-DM2)/(DM2*DM2))
      DLAMDA = DK/DM2
      DM = DSQRT(DM2)
      QZEXP = CDEXP (DCMPLX (0.0D0, -DLAMDA*XSTN))
      QEXP = DCMPLX(0.0D0,DLAMDA)
      J1 = J - 1
      U  =  9.0D-13
      ACC = 1.0D-6
      IF (IPT.GT.0) WRITE(6,990)DK,DR,DW,RHO,XSTN,A,B,J,IFT
  990 FORMAT('0',15X,'Q1DZXM ENTERED WITH ARGUMENTS:',/,
     1' ',15X,'DK',13X,'DR',13X,'DW    ',10X,'RHO',12X,'XSTN',11X,
     2'A',14X,'B',14X,'J',2X,'IPT',/,
     3' ',15X,7(E14.7,','),I2,2X,I3)
      EFCURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      QERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      QPSUM(LVL) = 0.0
      ALPHA = A
      DA = B - A
```

```
      AREA = 0.0
      AREST = 0.0
      FV(1) = F(ALPHA)
      FV(3) = F(ALPHA + 0.5*DA)
      FV(5) = F(ALPHA + DA)
      KOUNT = 3
      WT = DA/6.0
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))
1     DX = 0.5*DA
      FV(2) = F(ALPHA + 0.5*DX)
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFCURU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.60)GO TO 5
      IF(KOUNT.GE.4000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      QANS = ((B**J1)*CDEXP(QEXP*B) - QSUM)*Q2EXP/DN
      IF(IPT.GT.0) GO TO 11
      IF(IFLAG.EQ.1) RETURN
```

```
      WRITE(6,990) DK,DF,DW,RHO,XSTN,A,B,J,IPT                          QLDR6965
   11 WRITE(6,995) QANS,IFLAG,IER,QERRCR                                QLDR6970
  995 FORMAT(' ',15X,'Q1DZXM RESULTS: QANS = ',E14.7,',',E14.7,/,       QLDR6975
     1' ',15X,'IFLAG',2X,'IER',5X,'QERRCR',/,                           QLDR6980
     2' ',15X,I3,2X,I3,5X,E14.7,',',E14.7)                              QLDR6985
      RETURN                                                            QLDR6990
    4 CFSUM(LVL) = QSUM                                                 QLDR6995
      LCRR(LVL) = 1                                                     QLDR7000
      ALPHA = ALPHA + DA                                                QLDR7005
      DA = DAT(LVL)                                                     QLDR7010
      FV(1) = F1T(LVL)                                                  QLDR7015
      FV(3) = F2T(LVL)                                                  QLDR7020
      FV(5) = F3T(LVL)                                                  QLDR7025
      AREST = ARESTT(LVL)                                               QLDR7030
      QEST = QESTT(LVL)                                                 QLDR7035
      EPS = EPST(LVL)                                                   QLDR7040
      GO TO 1                                                           QLDR7045
    5 IFLAG = 2                                                         QLDR7050
      GO TO 2                                                           QLDR7055
    6 IFLAG = 3                                                         QLDR7060
      GO TO 2                                                           QLDR7065
      END                                                               QLDR7070
      COMPLEX FUNCTION Q1DAIP*16(DK,DR,DW,RHO,OFST,SGMA,XSTN,Q1CF,N,IPT)QLDR7075
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)                 QLDR7080
      DIMENSION Q1CF(13)                                                QLDR7085
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFST,SGMA,XSTN,IPT        QLDR7090
  990 FORMAT('0',10X,'Q1DAIP ENTERED WITH:',/,                          QLDR7095
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFST',9X,'SGMA',9XQLDR7100
     2  ,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)                      QLDR7105
      XASTN = XSTN - OFST                                               QLDR7110
      IF(XASTN.LE.DR+RHO-1.0D0) GO TO 20                                QLDR7115
      IOT = IPT - 1                                                     QLDR7120
      QCONST = CDEXP( DCMPLX(0.0D0,SGMA))                               QLDR7125
      A = DR-1.0D0                                                      QLDR7130
      B = XASTN - RHO - 1.0D-8                                          QLDR7135
      CALL Q1DZSM(DK,DR,DW,RHO,XSTN,A,B,N,QANS,Q1CF,IOT)                QLDR7140
      Q1DAIP = QANS                                                     QLDR7145
      IF(IPT.LE.0) RETURN                                               QLDR7150
      GOTO 60                                                           QLDR7155
   20 Q1DAIP = DCMPLX(0.0D0,0.0D0)                                      QLDR7160
      IF(IPT.LE.0)RETURN                                                QLDR7165
   60 WRITE (6,995) Q1DAIP                                              QLDR7170
  995 FORMAT('0',10X,'Q1DAIP = ',E14.7,',',E14.7)                       QLDR7175
      RETURN                                                            QLDR7180
      END                                                               QLDR7185
      COMPLEX FUNCTION Q1DRIP*16(DK,DR,DW,RHO,OFFSET,XSTN,Q1CF,N,IPT)   QLDR7190
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)                 QLDR7195
      DIMENSION Q1CF(13)                                                QLDR7200
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```
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1DRIF ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,
     2  'XSTN',9X,'IPT'/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.DR+RHO+OFFSET-1.0D0) GOTO 20
      ICT = IPT - 1
      A = DR-1.0D0
      B = XSTN - RHO - 1.0D-8
      CALL Q1DZSM(DK,DR,DW,RHO,XSTN,A,B,N,QANS,Q1CF,IOT)
      Q1DRIP = -QANS
      IF(IPT.LE.0) RETURN
      GOTO 60
   20 Q1DRIP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   60 WRITE (6,995) Q1DRIP
  995 FORMAT('0',10X,'Q1DRIP = ',E14.7,',',E14.7)
      RETURN
      END
      SUBROUTINE Q1DZSM (DK,DR,DW,RHO,XSTN,A,B,J,QANS,Q1CF,IPT)
      IMPLICIT REAL*8(A - E,G,H,M,O,P,R - Y), COMPLEX*16(F,Q,Z)
      DIMENSION Q1CF(13)
      DIMENSION FV(5),LORR(60),F1T(60),F2T(60),F3T(60),DAT(60),
     1  ARESTT(60),QESTT(60),EPST(60),QPSUM(60)
      F(X) = QLGNDR(X,DR,J,Q1CF)*CDEXP(QEXP*(X))
     1       * MMBSJ1((OMEGA * DSQRT( (XSTN-X)*(XSTN-X) - YY)), IER)
     2       * OMEGA * (XSTN-X)/(DSQRT((XSTN-X)*(XSTN-X)-YY))
      GAMMA = 1.4D0
      YY = RHO * RHO
      DM2 = (GAMMA + 1.0D0) * DW
      OMEGA = DSQRT (DK*DK*(1.0D0-DM2)/(DM2*DM2))
      DLAMDA = DK/DM2
      DM = DSQRT(DM2)
      Q2EXP = CDEXP (DCMPLX (0.0D0, -DLAMDA*XSTN))
      QEXP = DCMPLX(0.0D0,DLAMDA)
      U  =  9.0D-13
      ACC = 1.0D-6
      IF (IPT.GT.0) WRITE(6,990)DK,DR,DW,RHO,XSTN,A,B,J,IPT
  990 FORMAT('0',15X,'Q1DZSM ENTERED WITH ARGUMENTS:',/,
     1' ',15X,'DK',13X,'DR',13X,'DW    ',10X,'RHO',12X,'XSTN',11X,
     2'A',14X,'B',14X,'J',2X,'IPT',/,
     3' ',15X,7(E14.7,','),I2,2X,I3)
      EFCURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      QERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      QPSUM(LVL) = 0.0
```

```
      ALPHA = A                                                         QLDR7445
      DA = B - A                                                        QLDR7450
      AREA = 0.0                                                        QLDR7455
      AREST = 0.0                                                       QLDR7460
      FV(1) = F(ALPHA)                                                  QLDR7465
      FV(3) = F(ALPHA + 0.5*DA)                                         QLDR7470
      FV(5) = F(ALPHA + DA)                                             QLDR7475
      KOUNT = 3                                                         QLDR7480
      WT = DA/6.0                                                       QLDR7485
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))                             QLDR7490
    1 DX = 0.5*DA                                                       QLDR7495
      FV(2) = F(ALPHA + 0.5*DX)                                         QLDR7500
      FV(4) = F(ALPHA + 1.5*DX)                                         QLDR7505
      KOUNT = KOUNT + 2                                                 QLDR7510
      WT = DX/6.0                                                       QLDR7515
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))                            QLDR7520
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))                            QLDR7525
      QSUM = QESTL + QESTR                                              QLDR7530
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))      QLDR7535
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))      QLDR7540
      AREA = AREA + ((ARESTL + ARESTR) - AREST)                         QLDR7545
      QDIFF = QEST - QSUM                                               QLDR7550
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2                         QLDR7555
      IF(DABS(DX).LE.EPCURU*DABS(ALPHA))GO TO 5                         QLDR7560
      IF(LVL.GE.60)GO TO 5                                              QLDR7565
      IF(KOUNT.GE.4000)GO TO 6                                          QLDR7570
      LVL = LVL + 1                                                     QLDR7575
      LORR(LVL) = 0                                                     QLDR7580
      F1T(LVL) = FV(3)                                                  QLDR7585
      F2T(LVL) = FV(4)                                                  QLDR7590
      F3T(LVL) = FV(5)                                                  QLDR7595
      DA = DX                                                           QLDR7600
      DAT(LVL) = DX                                                     QLDR7605
      AREST = ARESTL                                                    QLDR7610
      ARESTT(LVL) = ARESTR                                              QLDR7615
      QEST = QESTL                                                      QLDR7620
      QESTT(LVL) = QESTR                                                QLDR7625
      EPS = EPS/1.4                                                     QLDR7630
      EPST(LVL) = EPS                                                   QLDR7635
      FV(5) = FV(3)                                                     QLDR7640
      FV(3) = FV(2)                                                     QLDR7645
      GO TO 1                                                           QLDR7650
    2 QERROR = QERROR + QDIFF/15.0                                      QLDR7655
    3 IF(LORR(LVL).EQ.0)GO TO 4                                         QLDR7660
      QSUM = QPSUM(LVL) + QSUM                                          QLDR7665
      LVL = LVL - 1                                                     QLDR7670
      IF(LVL.GT.1)GO TO 3                                               QLDR7675
      QANS = (QLGNDR(B,DR,J,Q1CF)*CDEXP(QEXP*B) - QSUM)*Q2EXP/DM        QLDR7680
```

```
      IF(IPT.GT.0) GO TO 11                                               QLDR7685
      IF(IFLAG.EQ.1) RETURN                                               QLDR7690
      WRITE(6,990) DK,DR,DW,RHO,XSTN,A,E,J,IPT                            QLDR7695
   11 WRITE(6,995) QANS,IFLAG,IER,QERROR                                  QLDR7700
  995 FORMAT(' ',15X,'QIDZSM RESULTS: QANS = ',E14.7,',',E14.7,/,         QLDR7705
     1' ',15X,'IFLAG',2X,'IER',5X,'QERROR',/,                             QLDR7710
     2' ',15X,I3,2X,I3,5X,E14.7,',',E14.7)                                QLDR7715
      RETURN                                                              QLDR7720
4     QFSUM(LVL) = QSUM                                                   QLDR7725
      LORR(LVL) = 1                                                       QLDR7730
      ALPHA = ALPHA + DA                                                  QLDR7735
      DA = DAT(LVL)                                                       QLDR7740
      FV(1) = F1T(LVL)                                                    QLDR7745
      FV(3) = F2T(LVL)                                                    QLDR7750
      FV(5) = F3T(LVL)                                                    QLDR7755
      AREST = ARESTT(LVL)                                                 QLDR7760
      QEST = QESTT(LVL)                                                   QLDR7765
      EPS = EPST(LVL)                                                     QLDR7770
      GO TO 1                                                             QLDR7775
5     IFLAG = 2                                                           QLDR7780
      GO TO 2                                                             QLDR7785
6     IFLAG = 3                                                           QLDR7790
      GO TO 2                                                             QLDR7795
      END                                                                 QLDR7800
```

7. Gorelov Spanning Function.

The subprogram for function used by Gorelov is shown below.

```
      REAL FUNCTION PLGNDR*8(X,DR,N)
      IMPLICIT REAL*8(A-H,O-Z)
      IF(N.EQ.0) GOTO 100
      ETA = DARCOS(-X)
      ETASTR = DARCOS(1.0D0 - DR)
      FN = DFLOAT(N)
      PLGNDR = DCOS(FN*ETA)-DCOS(FN*ETASTR)
      RETURN
  100 PLGNDR =  1.0D0
      RETURN
      END
```

8. Linear Expansion Program

The program based on the linear expansion for small k is shown below.

```
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Q,Z)
      DIMENSION X(13), Q2PT(13), Q2CP(13)
C     BLOCK ONE    READ AND EDIT DATA
      CALL ERRSET(208,256,-1,1)
      WRITE (6,910)
C
    1 CALL READ(DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT)
      IF(IPT.GT.0)WRITE(6,990)DK,DR,DW,SIGMA,OFFSET,RHO,N,NF
C
  910 FORMAT ('1',5X,'GORELOV CASCADE PROGRAM LINEARIZED FOR SMALL DK')
  990 FORMAT('0',5X,'INPUT VALUES TRANSMITTED TO MAIN PROGRAM:',/,
     1' ',10X,'DK',13X,'DR',13X,'DW    ',10X,'SIGMA',10X,'OFFSET',/,
     1' ',10X,5(E12.5,3X),/,
     2'0',10X,'RHO',12X,'N',5X,'NFCN'
     3,/,' ',10X,1(E12.5,3X), I2,4X,I2)
      CALL ABSA(N,OFFSET,X,RHO,DW    )
C
C
C     BLOCK TWO    CALLS CALCULATION ROUTINES FOR ZONE ONE AND ZONE TWO
      CALL Q$ONE(DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT,X)
      GO TO 1
      END
      SUBROUTINE READ (DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT)
      IMPLICIT REAL*8(A-H,O,P,R-Y)
      READ(5,905) DK,DR,DW,RHP,OFFSET,SIGMA,N,NF,IPT
      IF (DK.LT.0.0D0) STOP
      IF (IPT.GT.0) WRITE (6,910)
      GAMMA = 1.4D0
      DR = RHP * DSQRT((GAMMA + 1.0D0) * DW)
      RHO = DR
      IF(N.LE.2) GO TO  1
      N = 2
C
      WRITE(6,935 )
    1 IF(OFFSET.LE.1.9D0) GO TO 2
      WRITE(6,940)
      OFFSET = OFFSET-1.0D0
      GO TO 1
    2 CONTINUE
      IF (IPT.GT.0) WRITE (6,925) DK,DR,DW,SIGMA,OFFSET,RHO,N,NF,IPT
C
      RETURN
  905 FORMAT(6F10.4,3I2)
  910 FORMAT ('1',10X,'GORELOV SLIGHTLY SUPERSONIC CASCADE PROGRAM',/,
     *       ' ',15X,'LINEARIZED FOR SMALL FREQUENCY, DK.')
  925 FORMAT ('0',10X,'DK',13X,'DR',13X,'DW    ',10X,'SIGMA',10X,'OFFSET'
     1,/,' ',10X,5(E12.5,3X),/,
     2'0',10X,'RHO',12X,'N',5X,'NFCN',2X,'IPT'
     3,/,' ',10X,1(E12.5,3X), I2,4X,I2,4X,I2)
  935 FORMAT('0',5X,'ORIGINAL N TOO LARGE (.GT.13) - RESET TO N = 13')
  940 FORMAT('0',5X,'OFFSET TOO LARGE (.GT.1.9), RESET AS OFFSET =
     1OFFSET - 1.0D0')
```

```
      END
      SUBROUTINE ABSA (N,OFFSET,X,DR,DW    )
      IMPLICIT REAL*8 (A-H,C,P,R-Y), COMPLEX*16(Q,Z)
      DIMENSION X(13)
      XINT = (2.0D0-DR)/DFLOAT(N+1)
      XL = DR - 1.0D0 + 1.0D-8
      DO 10 I = 1,N
      X(I) = XL + (XINT*DFLOAT(I))
      IF(X(I).EQ.0.0D0) X(I) = .1D-14
   10 CONTINUE
      RETURN
      END
      SUBROUTINE Q$ONE(DK,DR,DW,RHO,OFFSET,SIGMA,N,NF,IPT,X)
      IMPLICIT REAL*8(A-H,C,P,R-Y), COMPLEX*16(Z,Q)
      DIMENSION Q1COF(26), Q1INT(26,26), Q1RBP(13), Q1ABP(13)
      DIMENSION Q1RBCF(13), Q1ABCF(13)
      DIMENSION X(13)
C     IF(IPT.GT.0)WRITE(6,990)DK,DR,RHO,OFFSET,SIGMA,N,NF,IPT
      ICT = IPT - 1
  990 FORMAT('0  SUBROUTINE Q$ONE ENTERED WITH:',/,
     1' ',5X,'DK',13X,'DR',13X,'RHO',12X,'OFFSET',9X,'SIGMA',10X,
     2'N',3X,'NF',2X,'IPT',/,
     3  5X,5(E12.5,3X),I2,2X,I2,2X,I3)
      CALL Q1ZINT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,ICT,Q1COF,Q1INT)
      CALL Q1COEF(Q1COF,Q1INT,N,IOT,Q1ABCF,Q1RBCF)
      CALL Q1PPOT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,Q1ABCF,Q1RBCF,ICT)
      I O T = 0
      CALL Q1DCOF(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IOT)
      RETURN
      END
C
C
C
      SUBROUTINE Q1ZINT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,
     1       IPT,Q1COF,Q1INT)
      IMPLICIT REAL * 8(A-H,O,P,R-Y), COMPLEX * 16(Z,Q)
      DIMENSION Q1COF(26), Q1INT(26,26), QINTRP(13), QINTAP(13)
      DIMENSION  X(13)
  990 FORMAT ('0',10X,'Q1ZINT ENTERED WITH:',/,
     1 ' ',10X,'DK',13X,'DR',13X,'RHO',12X,'OFFSET',9X,'SIGMA',10X,
     2  'N',2X,'IPT',/,
     3  ' ',10X,5(E12.5,3X),I2,2X,I3)
      QCONST = CDEXP(DCMPLX(0.0D0,SIGMA))
      IF (IPT.GT.0) WRITE (6,990) DK,DR,RHO,OFFSET,SIGMA,N,IPT
      ICT = IPT - 1
      GAMMA = 1.4D0
      CLAMDA = DK/((GAMMA+1.0D0)*DW)
      DO 90 I = 1,N
```

```
      IN = I + N
      XSTN = X(I)
      XC = XSTN - DR
      CEXP = CDEXP(DCMPLX(0.0D0,DLAMDA*XSTN))
      CALL Q1CAIP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,QINTAP,N,IOT)
      CALL Q1CRIP(DK,DR,DW,RHO,OFFSET,XSTN,QINTRP,N,IOT)
      DO 20 J = 1,N
       JN = N + J
      J1 = J-1
      Q1INT(I,1) = DCMPLX(1.0D0,DLAMDA*XSTN)
      TEMP = DR-XSTN-1.0D0
      Q1INT(I,2) = DCMPLX(TEMP,DLAMDA*XSTN*TEMP)
      Q1INT(IN,J) = QINTRP(J)
      Q1INT(I,JN) =  QINTAP(J)
      XASTN = XSTN - OFFSET
      Q1INT(IN,3) = DCMPLX(1.0D0,DLAMDA*XASTN)
      TEMP = DR-XASTN-1.0D0
      Q1INT(IN,4) = DCMPLX(TEMP,DLAMDA*XASTN*TEMP)
   20 CONTINUE
      Q1COF(IN) = -Q1CRBP(DK,DR,DW,RHO,XSTN,IOT)
      Q1COF(I)  = -Q1CABP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IOT)
   90 CONTINUE
      RETURN
      END
      COMPLEX FUNCTION Q1CRBP*16(DK,DR,DW,RHO,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION QINP(2)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1CRBP ENTERED WITH:',/,
     1 ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'XSTN',9X,'IPT'
     2 ,/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.RHO-1.0D0) GOTO 20
      IF(XSTN.GT.2.0D0) GOTO 20
      ICT = IPT - 1
      QDK = DCMPLX (0.0D0,DK)
      GAMMA = 1.4D0
      DLAMDA = DK/((GAMMA+1.0D0)*DW)
      Q1CRBP = DCMPLX(1.0D0,(DK+DLAMDA)*(XSTN-DR))
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 Q1CRBP = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0) RETURN
   30 WRITE (6,995) Q1CRBP
  995 FORMAT('0',10X,'Q1CRBP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1CABP*16(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
```

```
      DIMENSION QINP(2)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1CABP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,
     2  'SIGMA',8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFFSET
      IF(XASTN.LE.RHO-1.0D0) GOTO 20
      IF(XASTN.GT.2.0D0) GOTO 20
      IOT = IPT - 1
      CDK = DCMPLX (0.0D0,DK)
      QCONST = CDEXP( DCMPLX(0.0D0,SIGMA))
      GAMMA = 1.4D0
      DLAMDA = DK/((GAMMA+1.0D0)*DW)
      Q1CABP = DCMPLX(1.0D0,(DK+DLAMDA)*(XASTN-DR))*QCONST
      IF(IPT.LE.0) RETURN
      GO TO 30
   20 Q1CABP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   30 WRITE (6,995) Q1CABP
  995 FORMAT('0',10X,'Q1CABP = ',E14.7,',',E14.7)
      RETURN
      END
      SUBROUTINE Q1CAIP(DK,DR,DW,RHO,OFST,SGMA,XSTN,QCAIP,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION  QINP(13), QCAIP(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFST,SGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1CAIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFST',9X,'SGMA',9X
     2  ,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFST
      IF(XASTN.LE.DR+RHO-1.0D0) GO TO 20
      IF(XASTN.GT.2.0D0) GOTO 20
      IOT = IPT - 1
      QCONST = CDEXP( DCMPLX(0.0D0,SGMA))
      GAMMA = 1.4D0
      DLAMDA = DK/((GAMMA+1.0D0)*DW)
      QCAIP(1) = DCMPLX(1.0D0,DLAMDA*(XASTN-RHO))
      TEMP = DR+RHO-XASTN-1.0D0
      QCAIP(2)  = DCMPLX(TEMP,DLAMDA*(XASTN-RHO)*TEMP)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 ZERO = DCMPLX(0.0D0,0.0D0)
      DO 25 I = 1,N
      QCAIP(I) = ZERO
   25 CONTINUE
      IF(IPT.LE.0)RETURN
   30 WRITE (6,995)
  995 FORMAT ('0',10X,'Q1CAIP RESULTS: J      QCAIP(J)')
```

```
      DO 40 J = 1,N
      WRITE (6,996) J,QCAIP(J)
  996 FORMAT(' ',26X,I2,3X,E14.7,',',E14.7)
   40 CONTINUE
      RETURN
      END
      SUBROUTINE  Q1CRIP (DK,DR,DW,RHO,OFFSET,XSTN,QCRIP,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION  QCRIP(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1CRIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,
     2  'XSTN',9X,'IPT'/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.DR+RHO+OFFSET-1.0D0) GOTO 20
      IF(XSTN.GT.2.0D0) GOTO 20
      IOT = IPT - 1
      GAMMA = 1.4D0
      DLAMDA = DK/((GAMMA+1.0D0)*DW)
      QCRIP(1) = DCMPLX(1.0D0,DLAMDA*(XSTN-RHO))
      TEMP = DR+RHO-XSTN-1.0D0
      QCRIP(2)  = DCMPLX(TEMP,DLAMDA*(XSTN-RHO)*TEMP)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 ZERO = DCMPLX(0.0D0,0.0D0)
      DO 25 I = 1,N
   25 QCRIP(I) = ZERO
      IF (IPT.LE.0) RETURN
   30 WRITE (6,995)
  995 FORMAT ('0',10X,'Q1CRIP RESULTS: J     QCRIP(J)')
      DO 40 J = 1,N
      WRITE (6,996) J,QCRIP(J)
  996 FORMAT(' ',26X,I2,3X,E14.7,',',E14.7)
   40 CONTINUE
      RETURN
      END
      SUBROUTINE Q1COEF( Q1COF,Q1INT,N,IPT,Q1ABCF,Q1RBCF)
      IMPLICIT REAL * 8(A-H,O,P,R-Y), COMPLEX * 16 (Z,Q)
      DIMENSION Q1COF(26), Q1INT(26,26), ZWA(300)
      DIMENSION Q1ABCF(13),Q1RBCF(13)
      IF (IPT.GE.0) WRITE (6,90)
      IB = 26
      M = 1
      N2 = 2*N
      IF (IPT.LE.0) GO TO 5
      WRITE (6,98) N,N2
   98 FORMAT('0',5X,'Q1COEF ENTERED WITH ',I2,' DEG  PWR SERIES (',I2,
     1 ' SQUARE MATRIX)')
      DO 2 I = 1,N2
```

```
      WRITE(6,92) I, I, Q1COF(I)
   92 FORMAT('0',10X,'Q1COEF EQUATION SYSTEM, ROW ',I2,/,
     1' ',10X,'Q1COF(',I2,') = ', E14.7,',',E14.7)
      DO 2 J = 1,N
      J2=J+N
      WRITE(6,91) I,J,Q1INT(I,J),I,J2,Q1INT(I,J2)
   91 FORMAT(' ',15X,2('QINT(',I2,',',I2,') = ',E14.7,',',E14.7,10X))
    2 CONTINUE
    5 IA = 26
      IJOB=0
      CALL LEQT1C(Q1INT,N2,IA,Q1COF,M,IE,IJOB,ZWA,IER)
      IF(IER.EQ.0) GOTO 30
      IF (IER.EQ.129) GO TO 10
      WRITE(6,93)
   93 FORMAT( '0',10X,'Q1COEF - ITERATIVE IMPROVEMENT FAILED, MATRIX TOO
     1 ILL-CONDITIONED. USE RESULTS WITH CAUTION.')
      GO TO 30
   10 WRITE (6,95)
   95 FORMAT('0',10X,'Q1COEF - MATRIX ALGORITHMICALLY SINGULAR.  COEFFIC
     1IENTS SET TO ZERO.')
      ZERO = DCMPLX(0.0D0,0.0D0)
      DO 20 I = 1,N
      I2 = I+N
      Q1COF(I) = ZERO
      Q1COF(I2) = ZERO
   20 CONTINUE
   30 CONTINUE
      DO 35 I = 1,N
      IN = I + N
      Q1ABCF(I) = Q1COF(IN)
      Q1RBCF(I) = Q1COF(I)
   35 CONTINUE
      IF(IPT.LE.0) RETURN
      IF (IPT.GE.0) WRITE (6,94)
      DO 40 I = 1,N
      IM1 = I-1
      WRITE (6,99) I, IM1, Q1RBCF(I), Q1ABCF(I)
   40 CONTINUE
   99 FORMAT('0',5X,I2,10X,I2,10X,2(E14.7,5X,E14.7,10X))
   90 FORMAT( '0',10X,'SUBROUTINE Q1COEF -  COMPLEX POWER SERIES COEFFIC
     1IENTS')
   94 FORMAT( '0',5X,'INDEX',7X,'DEG POLY',4X,'REFERENCE BLADE TERMS Q1C
     3OF(INDEX)',7X,'ADJACENT BLADE TERMS Q1COF(2*INDEX)')
      RETURN
      END
      SUBROUTINE Q1PPOT(DK,DR,DW,RHO,OFFSET,SIGMA,N,X,Q1ABCF,Q1RBCF,IPT)
      IMPLICIT REAL * 8(A-H,O,P,R-Y), COMPLEX * 16(Z,Q)
      DIMENSION Q1ABCF(13), Q1RBCF(13), QRR(13), QAA(13), QPHI(13)
```

```
      DIMENSION QDRR(13),QDAA(13),QDPHI(13)
      DIMENSION X(13)
      IF(IPT.LE.0) RETURN
      WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
  990 FORMAT('1',10X,'Q1PPOT -CALCULATION OF COMPLEX POTENTIALS FOR ZONE
     1 ONE',/,'0',10X,'THESE CALCULATIONS ARE BASED ON THE SPANNING FUNC
     2TION COEFFICIENTS GIVEN DIRECTLY ABOVE',/
     3 ,' ',10X,'DK',13X,'DR',13X,'DK',13X,'RHO',12X,'OFFSET',9X,'SIGMA'
     4 ,10X,'N',3X,'IPT',/,' ',10X,
     56(E12.5,3X),I2,2X,I3)
   90 FORMAT ('0 X-STATION NUMBER ',I2,', XSTN = ',F6.4,/,
     1'  BL TOTAL POTENTIAL',14X,'REF BL POTENTIAL',8X,
     2 'ADJ BL POTENTIAL',8X,'REF BL INT POTENTIAL',5X,
     3 'ADJ BL INT POTENTIAL')
   91 FORMAT('  R  ',2(E14.7,','),8(E11.4,','))
   92 FORMAT('  A  ',2(E14.7,','),8(E11.4,','))
      ICT = IPT - 1
      RZERO = 0.0D0
      DO 10 I = 1,N
      XSTN = X(I)
      QRO = Q1PRBF(DK,DR,DW,RZERO,XSTN,IOT)
      QRP = Q1PRBP(DK,DR,DW,RHO,XSTN,ICT)
      QAO = Q1PABF(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,IOT)
      QAP = Q1PABF(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,ICT)
      QRIPO = Q1PRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)
      QRIPP = Q1PRIP(DK,DR,DW,RHO,OFFSET,XSTN,Q1RBCF,N,ICT)
      QAIPO = Q1PAIP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,Q1ABCF,N,ICT)
      QAIPP = Q1PAIP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,Q1ABCF,N,ICT)
      QR = QRO + QAP + QRIPO + QAIPP
      QA = QRP + QAO + QRIPP + QAIPO
      QPHI(I)=Q1PPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,XSTN,IOT)
      WRITE (6,90) I,XSTN
      WRITE(6,91) QR,QRO,QAP,QRIPO,QAIPP
      WRITE(6,92) QA,QRP,QAO,QRIPP,QAIPO
      QRR(I) = QR
      QAA(I) = QA
      QDRO = Q1DRBP(DK,DR,DW,RZERO,XSTN,IOT)
      QDRP = Q1DRBP(DK,DR,DW,RHO,XSTN,ICT)
      QDAO = Q1DABP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,IOT)
      QDAP = Q1DABP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,ICT)
      QDRIPO = Q1DRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,ICT)
      QDRIPP = Q1DRIP(DK,DR,DW,RHO,OFFSET,XSTN,Q1RBCF,N,IOT)
      QDAIPO = Q1DAIP(DK,DR,DW,RZERO,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)
      QDAIPP = Q1DAIP(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)
      QDR = QDRO + QDAP + QDRIPO + QDAIPP
      QDA = QDRP + QDAO + QDRIPP + QDAIPO
      QDPHI(I)=Q1DPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,
     1     XSTN,ICT)
```

```
      WRITE (6,95) I,XSTN
      WRITE(6,91) QDR,QDRO,QDAP,QDRIPO,QDAIPP
      WRITE(6,92) QDA,QDRP,QDAO,QDRIPF,QDAIPO
      QDRR(I) = QDR
      QDAA(I) = QDA
   95 FORMAT ('0 X-STATION NUMBER ',I2,', XSTN = ',F6.4,/,
     1'   BL TOTAL D(POT)/DX',14X,'REF BL D(POT)/DX',8X,
     2 'ADJ BL D(POT)/DX',8X,'REF BL INT D(POT)/DX',5X,
     3 'ADJ BL INT D(POT)/DX')
   10 CONTINUE
      WRITE(6,994)
  994 FORMAT ('1',10X,'SUMMARY LISTING',/,
     1'0',10X,'XSTN',7X,'SINGLE BLADE TOTAL POTENTIAL',6X,
     1  'REF BLADE POTENTIAL',15X,'ADJ BLADE POTENTIAL')
      DO 20 I = 1,N
      XSTN = X(I)
      WRITE(6,94) XSTN,QPHI(I),QRR(I),QAA(I)
   94 FORMAT(' ',10X,F6.4,3(5X,E14.7,',',E14.7))
   20 CONTINUE
      WRITE(6,996)
  996 FORMAT('0',10X,'XSTN',7X,'SINGLE BLADE TOTAL D(POT)/DX',6X,
     1  'REF BLADE D(POT)/DX',15X,'ADJ BLADE D(POT)/DX')
      DO 50 I = 1,N
      XSTN = X(I)
      WRITE(6,94) XSTN,QDPHI(I),QDRR(I),QDAA(I)
   50 CONTINUE
      RETURN
      END
      SUBROUTINE Q1DCOF(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION Q1ABCF(13), Q1RBCF(13)
      IF(IPT.GT.0)WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
  990 FORMAT('1',10X,'Q1DCOF - CALCULATION OF COMPLEX DIMENSIONLESS AERO
     1DYNAMIC COEFFICIENTS',/,
     3 '0',10X,'DK',13X,'DR',13X,'DW',13X,'RHO',12X,'OFFSET',9X,'SIGMA'
     4 ,10X,'N',3X,'IPT',/,' ',10X,
     56(E12.5,3X),I2,2X,I3)
      IOT = IPT - 3
  999 QDCL = Q1DCL(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IOT)
      QDCM = Q1DCM(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,IOT)
      GAMMA = 1.4D0
      TAU = (2.0D0*DSQRT((GAMMA + 1.0D0)*DW))/DR
      WRITE (6,90) DK,TAU,DW,N,SIGMA,QDCL,QDCM
   90 FORMAT('0',5X,'DK = ',F6.3,', TAU = ',F7.4,', DW = ',F6.3,', N = '
     1      ,I2,', SIGMA = ',F6.3,', CL = ',F9.4,',',F9.4,', CM = ',
     2      F9.4,',',F9.4)
      RETURN
      END
```

```
      COMPLEX FUNCTION Q1DCL*16(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBC
     1F,IPT)
      IMPLICIT REAL*8(A - E,G,H,O,P,R - Y), COMPLEX*16(F,Q,Z)
      DIMENSION Q1ABCF(13), Q1RBCF(13)
      DIMENSION FV(5),F1T(60),F2T(60),F3T(60),QESTT(60),QPSUM(60)
      DIMENSION DAT(60),  ARESTT(60),EPST(60)
      DIMENSION LORR(60)
      F(X) = (Q1DPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,ICT)
     1    -QALPHA*Q1PHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,ICT))
      IF (IPT.GT.0) WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
  990 FORMAT('0',10X,'Q1DCL ENTERED WITH:',/,
     3 ' ',10X,'DK',13X,'DR',13X,'DK',13X,'RHO',12X,'OFFSET',9X,'SIGMA'
     4 ,10X,'N',3X,'IPT',/,' ',10X,
     56(E12.5,3X),I2,2X,I3)
      A = - 1.0D0
      B = 1.0D0
      ICT = IPT - 1
      GAMMA = 1.4D0
      DLAMDA = DK/((GAMMA + 1.0D0) * DW)
      QALPHA = DCMPLX(0.0D0,DLAMDA-DK)
      U  =  9.0E - 13
      ACC = 1.0D-5
      EFCURU = 4.0*U
      IFLAG = 1
      EPS = ACC
      QERROR = DCMPLX(0.0D0,0.0D0)
      LVL = 1
      LORR(LVL) = 1
      QPSUM(LVL) = 0.0
      ALPHA = A
      DA = B - A
      AREA = 0.0
      AREST = 0.0
      FV(1) = F(ALPHA)
      FV(3) = F(ALPHA + 0.5*DA)
      FV(5) = F(ALPHA + DA)
      KOUNT = 3
      WT = DA/6.0
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))
    1 DX = 0.5*DA
      FV(2) = F(ALPHA + 0.5*DX)
      FV(4) = F(ALPHA + 1.5*DX)
      KOUNT = KOUNT + 2
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
```

```
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFCURU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.60)GO TO 5
      IF(KOUNT.GE.4000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      Q1DCL = QSUM * 2.0D0
      IF(IPT.GT.0) GO TO 11
      IF(IFLAG.EQ.1) RETURN
      WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
 11   WRITE(6,995) Q1DCL,IFLAG,QERROR
995   FORMAT(' ',15X,'RESULTS: Q1DCL = ',E14.7,',',E14.7,/,
     1' ',15X,'IFLAG',2X,'QERROR',/,
     2' ',15X,I3,2X,E14.7,',',E14.7)
      RETURN
4     QPSUM(LVL) = QSUM
      LORR(LVL) = 1
      ALPHA = ALPHA + DA
      DA = DAT(LVL)
      FV(1) = F1T(LVL)
      FV(3) = F2T(LVL)
      FV(5) = F3T(LVL)
      AREST = ARESTT(LVL)
      QEST = QESTT(LVL)
      EPS = EPST(LVL)
      GO TO 1
```

```
5     IFLAG = 2                                                         LIN04810
      GO TO 2                                                           LIN04820
6     IFLAG = 3                                                         LIN04830
      GO TO 2                                                           LIN04840
      END                                                               LIN04850
      COMPLEX FUNCTION Q1DCM*16(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCLIN04860
     1F,IPT)                                                            LIN04870
      IMPLICIT REAL*8(A - E,G,H,O,P,R - Y), COMPLEX*16(F,C,Z)          LIN04880
      DIMENSION Q1ABCF(13), Q1RBCF(13)                                  LIN04890
      DIMENSION FV(5),F1T(60),F2T(60),F3T(60),CESTT(60),CPSUM(60)      LIN04900
      DIMENSION DAT(60),  ARESTT(60),EPST(60)                           LIN04910
      DIMENSION LORR(60)                                                LIN04920
      F(X) = X*(Q1DPHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,ICT) LIN04930
     1    -QALPHA*Q1PHI(DK,DR,DW,RHO,OFFSET,SIGMA,N,Q1ABCF,Q1RBCF,X,ICT))LIN04940
      IF (IPT.GT.0) WRITE (6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT       LIN04950
990   FORMAT('0',10X,'Q1DCM ENTERED WITH:',/,                           LIN04960
     3 ' ',10X,'DK',13X,'DR',13X,'DK',13X,'RHO',12X,'OFFSET',9X,'SIGMA' LIN04970
     4 ,10X,'N',3X,'IPT',/,' ',10X,                                     LIN04980
     56(E12.5,3X),I2,2X,I3)                                             LIN04990
      A = - 1.0D0                                                       LIN05000
      B = 1.0D0                                                         LIN05010
      ICT = IPT - 1                                                     LIN05020
      GAMMA = 1.4D0                                                     LIN05030
      DLAMDA = DK/((GAMMA + 1.0D0) * DW)                                LIN05040
      QALPHA = DCMPLX(0.0D0,DLAMDA-DK)                                  LIN05050
      U  =  9.0E - 13                                                   LIN05060
      ACC = 1.0D-5                                                      LIN05070
      EPGURU = 4.0*U                                                    LIN05080
      IFLAG = 1                                                         LIN05090
      EPS = ACC                                                         LIN05100
      CERROR = DCMPLX(0.0D0,0.0D0)                                      LIN05110
      LVL = 1                                                           LIN05120
      LORR(LVL) = 1                                                     LIN05130
      CPSUM(LVL) = 0.0                                                  LIN05140
      ALPHA = A                                                         LIN05150
      DA = B - A                                                        LIN05160
      AREA = 0.0                                                        LIN05170
      AREST = 0.0                                                       LIN05180
      FV(1) = F(ALPHA)                                                  LIN05190
      FV(3) = F(ALPHA + 0.5*DA)                                         LIN05200
      FV(5) = F(ALPHA + DA)                                             LIN05210
      KOUNT = 3                                                         LIN05220
      WT = DA/6.0                                                       LIN05230
      QEST = WT*(FV(1) + 4.0*FV(3) + FV(5))                             LIN05240
1     DX = 0.5*DA                                                       LIN05250
      FV(2) = F(ALPHA + 0.5*DX)                                         LIN05260
      FV(4) = F(ALPHA + 1.5*DX)                                         LIN05270
      KOUNT = KOUNT + 2                                                 LIN05280
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```
      WT = DX/6.0
      QESTL = WT*(FV(1) + 4.0*FV(2) + FV(3))
      QESTR = WT*(FV(3) + 4.0*FV(4) + FV(5))
      QSUM = QESTL + QESTR
      ARESTL = WT*(CDABS(FV(1)) + CDABS(4.0*FV(2)) + CDABS(FV(3)))
      ARESTR = WT*(CDABS(FV(3)) + CDABS(4.0*FV(4)) + CDABS(FV(5)))
      AREA = AREA + ((ARESTL + ARESTR) - AREST)
      QDIFF = QEST - QSUM
      IF(CDABS(QDIFF).LE.EPS*DABS(AREA))GO TO 2
      IF(DABS(DX).LE.EFOLRU*DABS(ALPHA))GO TO 5
      IF(LVL.GE.60)GO TO 5
      IF(KOUNT.GE.4000)GO TO 6
      LVL = LVL + 1
      LORR(LVL) = 0
      F1T(LVL) = FV(3)
      F2T(LVL) = FV(4)
      F3T(LVL) = FV(5)
      DA = DX
      DAT(LVL) = DX
      AREST = ARESTL
      ARESTT(LVL) = ARESTR
      QEST = QESTL
      QESTT(LVL) = QESTR
      EPS = EPS/1.4
      EPST(LVL) = EPS
      FV(5) = FV(3)
      FV(3) = FV(2)
      GO TO 1
2     QERROR = QERROR + QDIFF/15.0
3     IF(LORR(LVL).EQ.0)GO TO 4
      QSUM = QPSUM(LVL) + QSUM
      LVL = LVL - 1
      IF(LVL.GT.1)GO TO 3
      Q1DCM = QSUM * 2.0D0
      IF(IPT.GT.0) GO TO 11
      IF(IFLAG.EQ.1) RETURN
      WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,N,IPT
  11  WRITE(6,995) Q1DCM,IFLAG,QERROR
 995  FORMAT(' ',15X,'RESULTS: Q1DCM = ',E14.7,',',E14.7,/,
     1' ',15X,'IFLAG',2X,'QERROR',/,
     2' ',15X,I3,2X,E14.7,',',E14.7)
      RETURN
4     QPSUM(LVL) = QSUM
      LORR(LVL) = 1
      ALPHA = ALPHA + DA
      DA = DAT(LVL)
      FV(1) = F1T(LVL)
      FV(3) = F2T(LVL)
```

```
      FV(5) = F3T(LVL)
      AREST = ARESTT(LVL)
      QEST = QESTT(LVL)
      EPS = EPST(LVL)
      GO TO 1
    5 IFLAG = 2
      GO TO 2
    6 IFLAG = 3
      GO TO 2
      END
      COMPLEX FUNCTION Q1PHI*16 (DK,DR,DW,RHO,OFFSET,SIGMA,
     1    N,Q1ABCF,Q1RBCF,XSTN,IPT)
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Q,Z)
      DIMENSION Q1ABCF(13), Q1RBCF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1PHI ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,'SIGMA'
     2  ,8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      RZERO = 0.0D0
      IOT = IPT - 1
      Q1PHI = Q1PRBP(DK,DR,DW,RZERO,XSTN,IOT)
     1 + Q1PRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)
     2 + Q1PABP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,IOT)
     3 + Q1PAIP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)
      IF(IPT.LE.0) RETURN
      WRITE (6,995) Q1PHI
  995 FORMAT('0',10X,'Q1PHI = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1PRBP*16(DK,DR,DW,RHO,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION QINP(2)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1PRBP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'XSTN',9X,'IPT'
     2  ,/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.RHO-1.0D0) GOTO 20
      IOT = IPT - 1
      GAMMA = 1.4D0
      DM2 = (GAMMA + 1.0D0) * DW
      DLAMDA = DK/DM2
      PR = 1.0+XSTN-RHO
      PIMAG = (DK+DLAMDA)*((XSTN-RHO)*(XSTN-RHO) - 1.0D0)
      Q1PRBP = - DCMPLX(PR,PIMAG-DLAMDA*XSTN*PR)/DSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GOTO 60
   20 Q1PRBP = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0) RETURN
```

```
   60 WRITE (6,995) Q1PRBP
  995 FORMAT('0',10X,'Q1PRBP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1PABP*16(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION QINP(2)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1PABP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,
     2  'SIGMA',8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFFSET
      IF(XASTN.LE.RHO-1.0D0) GOTO 20
      ICT = IPT - 1
      CCONST = CDEXP( DCMPLX(0.0D0,SIGMA))
      GAMMA = 1.4D0
      DM2 = (GAMMA + 1.0D0) * DW
      DLAMDA = DK/DM2
      PR = 1.0D0 + XASTN - RHO
      PIMAG = (DK+DLAMDA)*((XASTN-RHO)*(XASTN-RHO) - 1.0D0)
      Q1PABP =  DCMPLX(PR,PIMAG-DLAMDA*XASTN*PR)*(CCONST/DSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GO TO 60
   20 Q1PABP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   60 WRITE (6,995) Q1PABP
  995 FORMAT('0',10X,'Q1PABP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1DPHI*16 (DK,DR,DW,RHO,OFFSET,SIGMA,
     1     N,Q1ABCF,Q1RBCF,XSTN,IPT)
      IMPLICIT REAL*8(A-H,O,P,R-Y), COMPLEX*16(Q,Z)
      DIMENSION Q1ABCF(13),Q1RBCF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1DPHI ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,'SIGMA'
     2  ,8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      RZERO = 0.0D0
      ICT = IPT - 1
      Q1DPHI = Q1DRBP(DK,DR,DW,RZERO,XSTN,IOT)
     1 + Q1DRIP(DK,DR,DW,RZERO,OFFSET,XSTN,Q1RBCF,N,IOT)
     2 + Q1DABP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,IOT)
     3 + Q1DAIP(DK,DR,DW,DR,OFFSET,SIGMA,XSTN,Q1ABCF,N,IOT)
      IF(IPT.LE.0) RETURN
      WRITE (6,995) Q1DPHI
  995 FORMAT('0',10X,'Q1DPHI = ',E14.7,',',E14.7)
      RETURN
      END
```

```
      COMPLEX FUNCTION Q1DRBP*16(DK,DR,DW,RHO,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION DINP(2)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1DRBP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'XSTN',9X,'IPT'
     2  ,/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.RHO-1.0D0) GOTO 20
      ICT = IPT - 1
      GAMMA = 1.4D0
      DM2 = (GAMMA + 1.0D0) * DW
      DLAMDA = DK/DM2
      DM2 = (GAMMA + 1.0D0) * DW
      PIMAG =  (DK+DLAMDA)*(XSTN-RHO)
      Q1DRBP = -DCMPLX(1.0D0,PIMAG-DLAMDA*XSTN)/DSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GOTO 60
   20 Q1DRBP = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0) RETURN
   60 WRITE (6,995) Q1DRBP
  995 FORMAT('0',10X,'Q1DRBP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1DABP*16(DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT)
      IMPLICIT REAL * 8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION DINP(2)
      IF(IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFFSET,SIGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1DABP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFFSET',7X,
     2  'SIGMA',8X,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFFSET
      IF(XASTN.LE.RHO-1.0D0) GOTO 20
      ICT = IPT - 1
      QCCNST = CDEXP( DCMPLX(0.0D0,SIGMA))
      GAMMA = 1.4D0
      DM2 = (GAMMA + 1.0D0) * DW
      DLAMDA = DK/DM2
      PIMAG =  (DK+DLAMDA)*(XASTN-RHO)
      Q1DABP =  DCMPLX(1.0D0,PIMAG-DLAMDA*XASTN)*QCCNST/DSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GO TO 60
   20 Q1DABP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   60 WRITE (6,995) Q1DABP
  995 FORMAT('0',10X,'Q1DABP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1PAIP*16(DK,DR,DW,RHO,OFST,SGMA,XSTN,Q1CF,N,IPT)
```

```
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION Q1CF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFST,SGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1PAIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFST',9X,'SGMA',9X
     2  ,'XSTN',9X,'IPT'/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFST
      IF(XASTN.LE.DR+RHO-1.0D0) GO TO 20
      QCONST = CDEXP( DCMPLX(0.0D0,SGMA))
      GAMMA = 1.4D0
      DM2 = (GAMMA+1.0D0)*DW
      DLAMDA = DK/DM2
      XX=XASTN-RHO
      Q1 = Q1CF(1)
      Q2 = Q1CF(2)
      PR1 = XX+1.0D0-DR
      PR2 = (DR-1.0D0 - XX/2.0D0)*XX - (DR-1.0D0)*(DR-1.0D0)/2.0D0
      PIMAG1 = (XX*XX+(2.0D0-DR)*DR-1.0D0)*DLAMDA/2.0D0
      PIMAG2 =  XX*XX*((DR-1.0D0)/2.0D0-XX)-((DR-1.0D0)**3)/6.0D0
      PIMAG1=PIMAG1*DLAMDA
      PIMAG2 = PIMAG2 * DLAMDA
      QP1 = DCMPLX(PR1,PIMAG1-DLAMDA*PR1*XASTN)
      QP2 = DCMPLX(PR2,PIMAG2-DLAMDA*PR2*XASTN)
      Q1PAIP = (Q1*QP1 + Q2*QP2)/CSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 Q1PAIP = DCMPLX(0.0D0,0.0D0 )
      IF(IPT.LE.0)RETURN
   30 WRITE (6,995) Q1PAIP
  995 FORMAT('0',10X,'Q1PAIP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1PRIP*16(DK,DR,DW,RHO,OFFSET,XSTN,Q1CF,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION Q1CF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1PRIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,
     2  'XSTN',9X,'IPT'/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.DR+RHO+OFFSET-1.0D0) GOTO 20
      GAMMA = 1.4D0
      DM2 = (GAMMA+1.0D0)*DW
      DLAMDA = DK/DM2
      XX = XSTN - RHO
      Q2 = Q1CF(2)
      Q1 = Q1CF(1)
      PR1 = XX+1.0D0-DR
      PR2 = (DR-1.0D0 - XX/2.0D0)*XX - (DR-1.0D0)*(DR-1.0D0)/2.0D0
```

```
      PIMAG1 = (XX*XX+(2.0D0-DR)*DR-1.0D0)*DLAMDA/2.0D0
      PIMAG2 =   XX*XX*((DR-1.0D0)/2.0D0-XX)-((DR-1.0D0)**3)/6.0D0
      PIMAG1=PIMAG1*DLAMDA
      PIMAG2 = PIMAG2 * DLAMDA
      QP1 = DCMPLX(PR1,PIMAG1-DLAMDA*PR1*XSTN)
      QP2 = DCMPLX(PR2,PIMAG2-DLAMDA*PR2*XSTN)
      Q1PRIP =-(Q1*QP1+Q2*QP2)/DSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 Q1PRIP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   30 WRITE (6,995) Q1PRIP
  995 FORMAT('0',10X,'Q1PRIP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1DAIP*16(DK,DR,DW,RHO,OFST,SGMA,XSTN,Q1CF,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
      DIMENSION Q1CF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,OFST,SGMA,XSTN,IPT
  990 FORMAT('0',10X,'Q1DAIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,'OFST',9X,'SGMA',9X
     2  ,'XSTN',9X,'IPT',/,' ',10X,7(E12.5,','),I3)
      XASTN = XSTN - OFST
      IF(XASTN.LE.DR+RHO-1.0D0) GO TO 20
      QCONST = CDEXP( DCMPLX(0.0D0,SGMA))
      GAMMA = 1.4D0
      DM2 = (GAMMA+1.0D0)*DW
      DLAMDA = DK/DM2
      XX = XASTN - RHO
      Q2 = Q1CF(2)
      Q1 = Q1CF(1)
      PR1 = 1.0D0
      PR2 = DR-1.0D0-XX
      PIMAG1 = DLAMDA*XX
      PIMAG2 =DLAMDA*(DR-1.0D0-XX)*XX*XX
      QP1 = DCMPLX(PR1,PIMAG1-DLAMDA*PR1*XASTN)
      QP2 = DCMPLX(PR2,PIMAG2-DLAMDA*PR2*XASTN)
      Q1DAIP = (Q1*QP1 + Q2*QP2)/DSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 Q1DAIP = DCMPLX(0.0D0,0.0D0)
      IF(IPT.LE.0)RETURN
   30 WRITE (6,995) Q1DAIP
  995 FORMAT('0',10X,'Q1DAIP = ',E14.7,',',E14.7)
      RETURN
      END
      COMPLEX FUNCTION Q1DRIP*16(DK,DR,DW,RHO,OFFSET,XSTN,Q1CF,N,IPT)
      IMPLICIT REAL*8 (A-H,O,P,R-Y), COMPLEX * 16 (Q,Z)
```

```
      DIMENSION Q1CF(13)
      IF (IPT.GT.0) WRITE(6,990) DK,DR,DW,RHO,XSTN,IPT
  990 FORMAT('0',10X,'Q1DRIP ENTERED WITH:',/,
     1  ' ',10X,'DK',11X,'DR',11X,'DW',11X,'RHO',10X,
     2  'XSTN',9X,'IPT'/,' ',10X,5(E12.5,','),I3)
      IF(XSTN.LE.DR+RHO+OFFSET-1.0D0) GOTO 20
      GAMMA = 1.4D0
      DM2 = (GAMMA+1.0D0)*DW
      DLAMDA = DK/DM2
      Q1 = Q1CF(1)
      Q2 = Q1CF(2)
      XX = XSTN - RHO
      PR1 = 1.0D0
      PR2 = DR-1.0D0-XX
      PIMAG2 =DLAMDA*(DR-1.0D0-XX)*XX*XX
      PIMAG1 = DLAMDA*XX
      QP1=DCMPLX(PR1,PIMAG1-DLAMDA*PR1*XSTN)
      QP2 = DCMPLX(PR2,PIMAG2-DLAMDA*PR2*XSTN)
      Q1DRIP =-(Q1*QP1+Q2*QP2)/CSQRT(DM2)
      IF(IPT.LE.0) RETURN
      GOTO 30
   20 Q1DRIP = DCMPLX(0.0D0,0.0D0)
      IF (IPT.LE.0) RETURN
   30 WRITE (6,995) Q1DRIP
  995 FORMAT('0',10X,'Q1DRIP = ',E14.7,',',E14.7)
      RETURN
      END
```

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